Single + Double Slit Interference.
2024 version

Purpose (1) Record and fit the single slit diffraction pattern.
(2) Record and plot data for the double slit diffraction pattern.
For (1), use the laser wavelength to determine a value for the slit width.

Optional: fit the double slit pattern + determine values for the slit width + separation.

Theory

The relevant theory is developed in numerous texts. Only the final results are quoted here with a description of each parameter. The variables are:

- θ : angle of deviated rays which have been diffracted by the slit.
- a : the width of the slit for the single slit
- d : the separation between adjacent slits.
- λ : the wavelength of the light source (laser)
- b : the width of each slit for a double slit setup.
- x : linear distance on a screen
- l : distance from slit(s) to screen.

For the single slit, the intensity of light on the screen will follow:

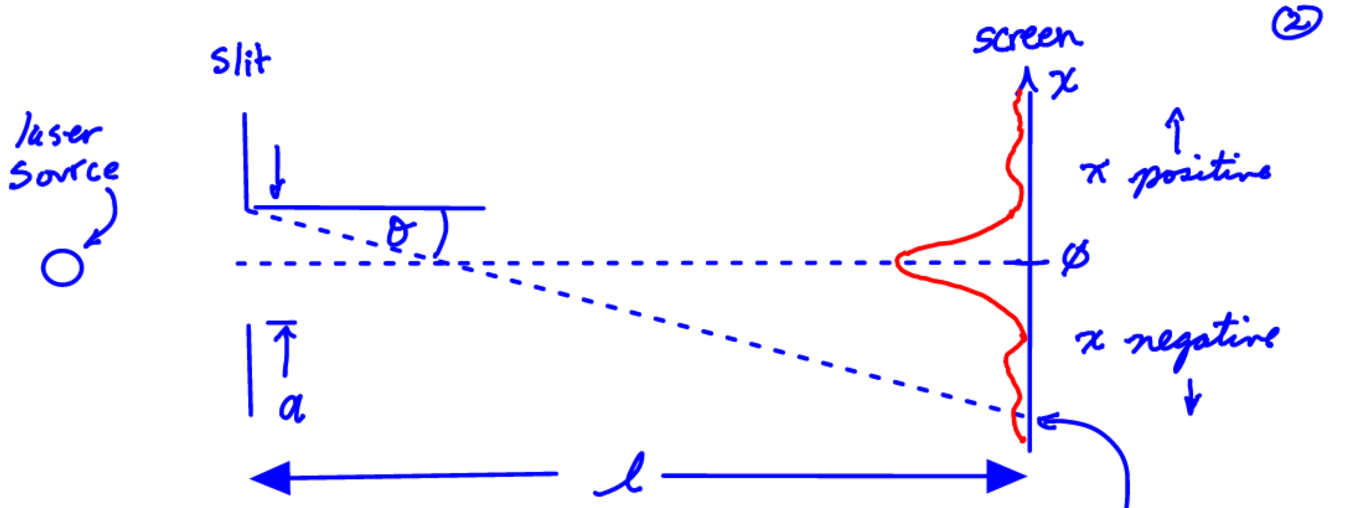
$$I(\theta) = I_0 \frac{\sin^2\beta}{\beta^2}$$

where:

$$\beta = \frac{\pi a \sin\theta}{\lambda}$$ (See figure on top of next page)

For small angles of θ, the linear position on the screen will scale in the same way (doubling θ will double x), so to a good approximation

$$I(x) = I_0 \frac{\sin^2\beta}{\beta^2}$$



Double Slit Equation

If a double slit is used, the relevant intensity function is

$$I(x) = 4I_0 \frac{\sin^2\beta}{\beta} \cos^2\alpha \qquad (\beta \text{ is as before})$$

where $\alpha = \dfrac{\pi d \sin\theta}{\lambda}$ and d = slit separation.

Note that this formula merely imposes the multipliers four and $\cos^2\alpha$ onto the single slit formula.

Apparatus:

Pasco laser diode, 1.2m optics bench, single slit accessory, double slit accessory, regular or high sensitivity light sensor, rotary motion sensor, linear translator

Procedure: Mounting Components on the Bench.

Set the equipment up as shown in Figure 1.

Table 1 has approximate positions.

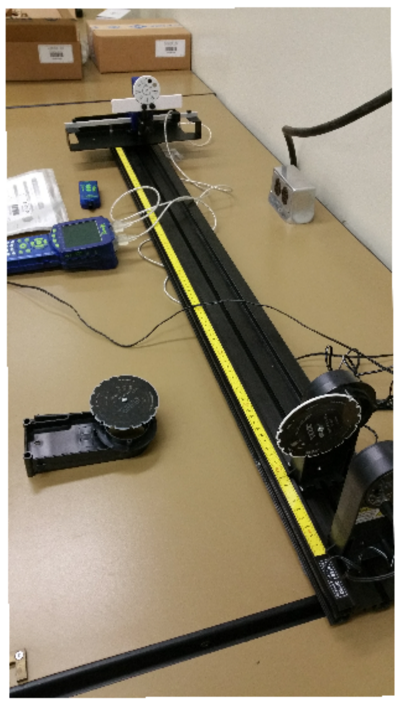
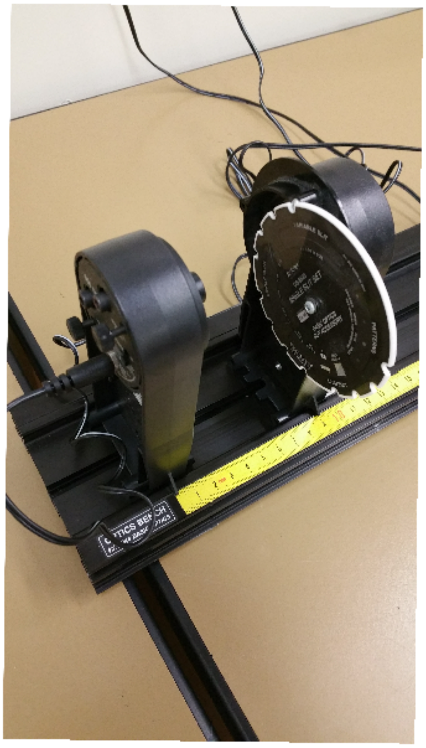


Figure 1. Apparatus

laser source	~ 0.0 cm
slit accessory	~ 7.0 cm
slit aperture	~ 100 cm
front edge of linear tr.	~

Table 1. Positions

See the following figures 2a + 2b for more detail:



"L" is the distance from the slit wheel here, to the aperture slits here

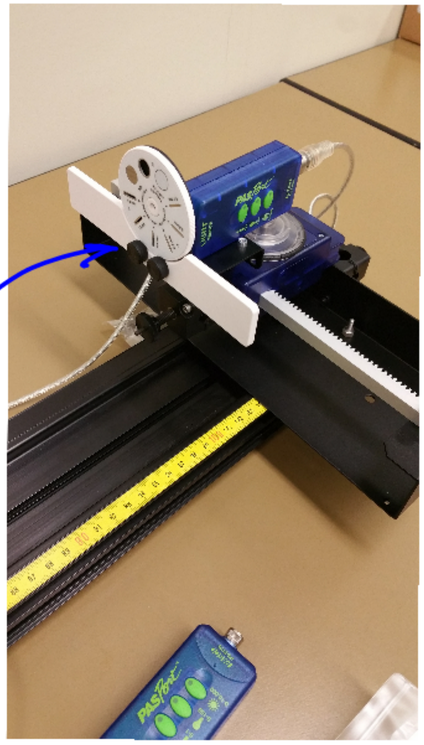


Figure 2a

Figure 2b

The slit accessory uses a mount which allows rotation and translation due to two offset axis of rotation. Get familiar with this behavior. The orientation of the linear translator requires a diffraction pattern which spreads out horizontally as shown in Figure 3.

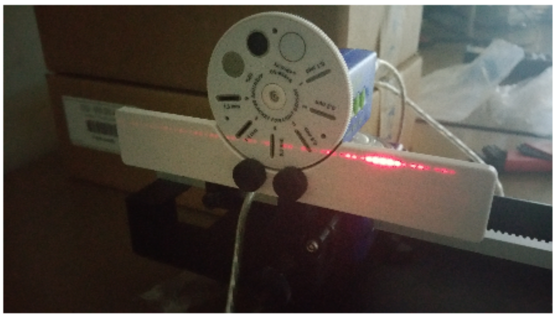


Figure 3: Single slit diffraction pattern. Rotate the slit accessory for height and alignment to achieve a pattern that is horizontal + centered on the slit in front of the light sensor.

Procedure: Choice of Slits

The slit accessory wheel sets the distance "a". The aperture (wheel) in front of the light sensor boosts resolution: only a small section detected. This second slit is chosen to give good sensitivity to changes in detected light intensity without saturating the detector readings (flat top). Record your values here:

Pattern	Slit Wheel a or a,b	Aperture slit #	Light sensor range
Single slit			
Double slit			
Laser λ =			

Table 2: slit + sensor settings

Procedure: Taking Data.

2023 Use "rack" in RMS sensor settings.

After getting a reasonably bright + level pattern projected onto the slit aperture, record a pattern.

Data collection must start to the left or right of the pattern (not the center).

Set X to Linear Position (=RMS rack) m. Set Y to Intensity in lux. Use Scientific Notation, 4 digits for both.

Start just to the outside of a faint maxima. Press record on the GLX and slowly turn the pulley wheel to make the light sensor translate across the pattern. Steadily move the sensor. Practice this until you record a reasonably good pattern.

Procedure: Double Slit Pattern

Switch to the double slit accessory wheel.
Record a, b, l.

Suggested double slit:
a = 0.08 mm = slit width.
d = 0.25 mm = slit separation.

Analysis:

The analysis consists of two main steps:

① Plot the sensor data in xmgrace
② Fit the data
③ Determine slit width (and/or slit width + separation).

Export the "best" single and double slit runs from the GLX to a USB stick:

Home, Tables, ✓, arrow to Run#, ✓, choose the run to export.

Populate two columns with linear position ← x & light intensity ← y.

This will create a text file with three columns since the row index number will be in the left-most column. Importing into xmgrace will involve choosing only two columns: X from column 2 and Y from column 3. Import using "Block data"

Import data to xmgrace: Single Slit

Data, Import, ASCII, Load As: Block Data, OK. Choose the columns which have linear position for X and light intensity for Y.

Now that the data is available as a set in xmgrace, eg g0.s0 with x as linear position and y as the light intensity,

A fit using non-linear curve fitting in xmgrace will be generated

It is likely that your data is not centered about $x = 0$. The theoretical equation assumes 0 is @ the center + @ the max I. A correction to the observed data is one strategy: <u>shift the recorded data so that the central maximum is $x = 0$ m.</u>

Step #1:

Center the observed peak to zero linear displacement to create a new set which is centered on zero:

① Scan down the data in the xmgrace's table g0.s0 and locate the maximum light intensity value. (Careful:

did the light sensor capture this point?)

② Add (or subtract) this offset to the linear position of a new set eg. g0.s1.x ; the Y will be the same values: g0.s0.y.

③ Hide the original set, scale + display this new set.

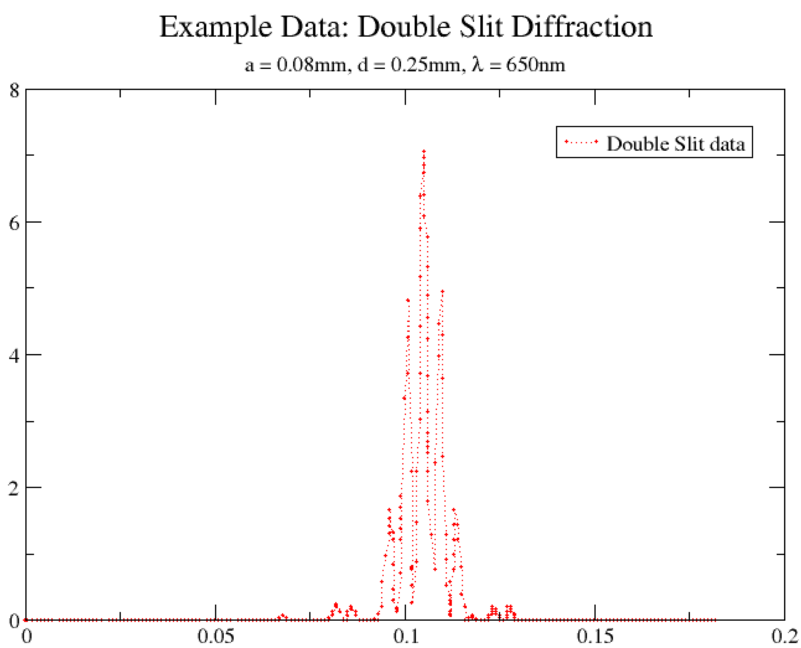
③ Label the new set : "centeredpeak" in the set comment field and display the set comments.

Import Double Slit Data

Use the Block data import feature of xmgrace to import the double slit data. An example is shown on the next page. The dashed line is not a theoretical fit but merely a dashed line joining the data points.

Question:

What further step would be required to predict the double slit pattern with reference to the steps taken to plot the single slit pattern?



Note: You'll likely get more interesting results!

Analysis: Hints + Helps

Below is just a copy of the theory from previous pages:

- θ: angle of deviated rays which have been diffracted by the slit.
- a: the width of the slit for the single slit
- d: the separation between adjacent slits.
- λ: the wavelength of the light source (laser)
- b: the width of each slit for a double slit setup.
- x: linear distance on a screen
- l: distance from slit(s) to screen.

For the single slit, the intensity of light on the screen will follow:

$$I(\theta) = I_0 \frac{\sin^2\beta}{\beta^2}$$

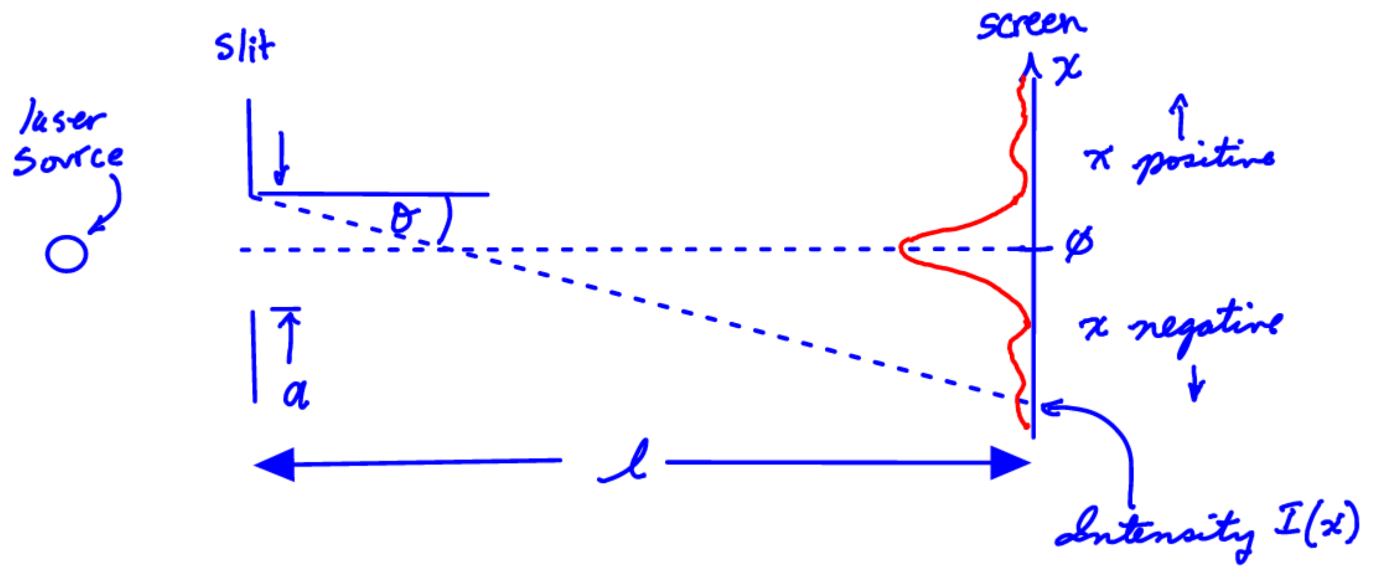
where:

$$\beta = \frac{\pi a \sin\theta}{\lambda}$$

For small angles of θ, the linear position on the screen will scale in the same way (doubling θ will double x), so to a good approximation in:

$$I(x) = I_0 \frac{\sin^2\beta}{\beta^2}$$

This allows a simpler β term which is helpful.



From the diagram above, $\sin\theta = x/l \Rightarrow \theta = \sin^{-1} x/l$. Therefore the equation for I_0 when expressed in x + not θ changes.

Now we have:

$$I = I_0 \left(\frac{\sin\left(\frac{\pi a}{\lambda} \cdot x/\ell\right)}{\frac{\pi a}{\lambda} \cdot x/\ell} \right)^2$$

using $\theta = x/\ell$ not $\theta = \sin^{-1} x/\ell$.

Discussion:

If we measure ℓ, we could use an xmgrace fit to the data to determine the slit width "a", for example. We also would provide λ.

Or we could measure or state ℓ + a, we could get λ from the fit.

The data must be shifted so that the peak I is @ $x = \phi$ or*

I_0 would have to be provided to xmgrace as the peak value on the graph. That value is about 1.5 lux on the plot. λ is 650e-9m. You will need to look @ your dataset to get a value of your peak I. ℓ = 100.0 cm − 7.0 cm = 93e-2 m ... These values when fed in would allow one to determine the slit width a after you have centered the peak @ $x = \phi$ m

$$y = 1.5\ \text{lux} \left(\frac{\sin\left[\frac{\pi}{650e\text{-}9m} \cdot A_1 \cdot \frac{x}{93.0e\text{-}2m}\right]}{\pi \cdot A_1/650e\text{-}9m \cdot \frac{x}{93.0e\text{-}2m}} \right)^2$$ ... Eq'n for plotting

2024: skip this

put your I_0 here is your max I#

2024 Fitting:

Allowing xmgrace the freedom to raise + lower the fit will be taken care of by A_0. A_1 will yield a value for the slit width a:

So the non-linear fit equation is:

$$\boxed{y = A_0 + 1.5 \cdot \left(\sin(5.1970e6 * A_1 * x) / (5.1970e6 * A_1 * x) \right)^2}$$

Details of fit parameters for non-linear xmgrace fit:

i) With A_0 + A_1 not bounded a fit is produced but it is a horizontal line @ the average intensity I

ii) Bounding A_1 to near the given slit width for the pattern (0.08mm = 8e-5 m) will assist producing a fit. This yielded:

$$A_0 = 0.00844\ \text{lux}$$
$$A_1 = 7.56e\text{-}5\ m$$

The A_1 agrees well with the Pasco value of a = 0.08mm. A plot with this fit is shown on the next page.

* use $(x + A_3)$ for x in the fit equation.

2020 Updates.

Below is just a copy of the theory from previous page.

θ : angle of deviated rays which have been diffracted by the slit.
a : the width of the slit for the single slit
d : the separation between adjacent slits.
λ : the wavelength of the light source (laser)
b : the width of each slit for a double slit setup.
x : linear distance on a screen
ℓ : distance from slit(s) to screen.

For the single slit, the intensity of light on the screen will follow:

$$I(\theta) = I_0 \frac{\sin^2\beta}{\beta^2}$$

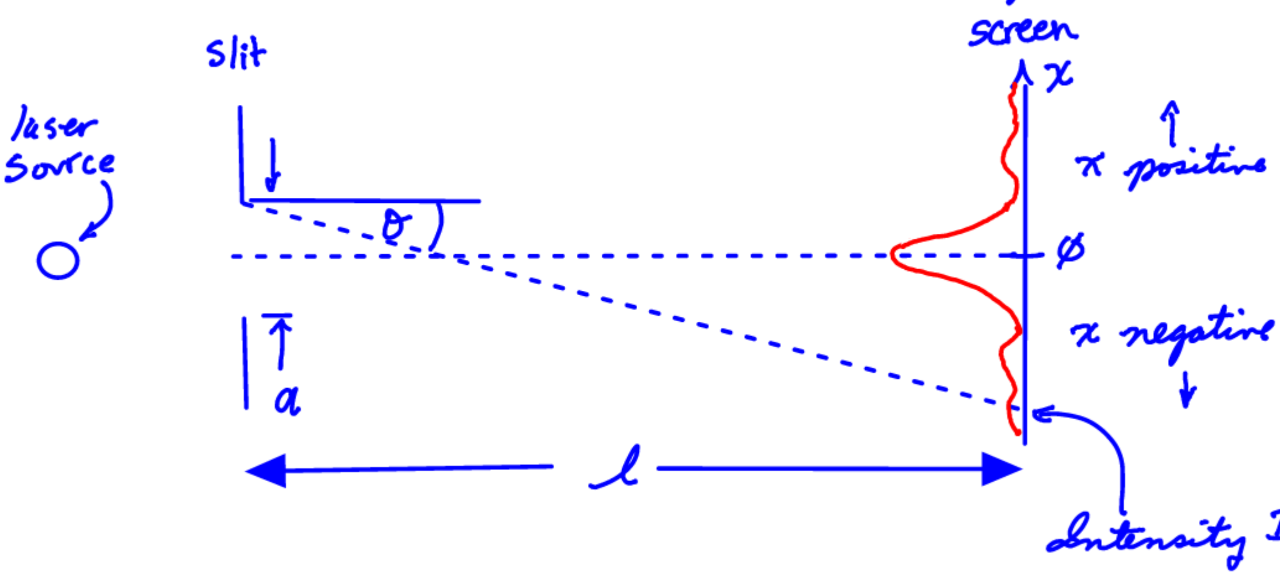
where:

$$\beta = \frac{\pi a \sin\theta}{\lambda}$$ (See figure on top of next page)

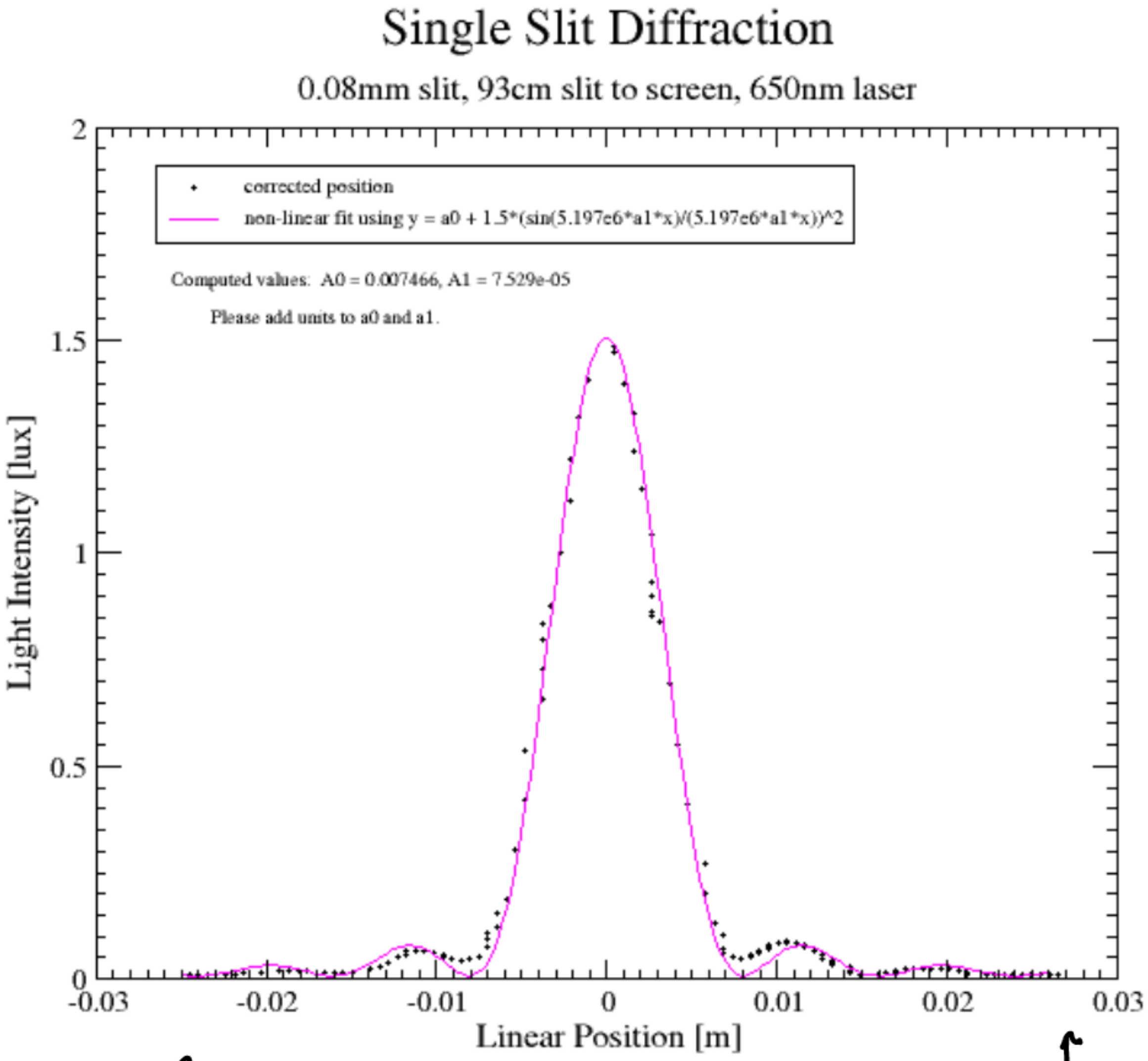
For small angles of θ, the linear position on the screen will scale in the same way (doubling θ will double x), so to a good approximation in:

$$I(x) = I_0 \frac{\sin^2\beta}{\beta^2}$$

This allows a simpler β term which is helpful.



From the diagram above, $\sin\theta = x/\ell \Rightarrow \theta = \sin^{-1} x/\ell$. Therefore the equation for I_0 when expressed in x + not θ changes.



2020 has new data, this is an example. from 2016.

-0.25 *

0.26 +

The above plot used the following settings on the Advanced tab in the Non-linear curve fitting dialog:

Load: Function

Start load at: -0.025 Stop load at 0.026 # of points 200

* shown above

This will show a smooth curve produced by the fit equation. Xmgrace will produce + plot 200 points in the fit set.

Double Slit Analysis: Fit with xmgrace

A similar effort can be made to fit the double slit data. The parameters are λ, ℓ, b = slit width + d = slit separation. The relevant formula for I (position θ) is:

$$I(x) = 4 I_0 \frac{\sin^2\beta}{\beta} \cos^2\alpha$$

$$\alpha = \frac{\pi d \sin\theta}{\lambda}$$ ← d = slit separation.

and β remains the same.

Again $\sin\theta = x/\ell$

$$\beta = \frac{\pi a \sin\theta}{\lambda}$$ our "b" value this time for dbl slit.

The 2020 pattern used $b = 0.04$ mm + $d = 0.50$ mm.
For the 2020 data, $\ell = 0.915$ m (different from single slit trial)

Let's seek a value for the slit separation "d" this time.

Thus using $k = \frac{\pi}{\lambda \ell} = 5.2822e6\ m^{-2}$

$$I(x) = 4 I_0 * (\sin[k\, a \cdot x]/k\, a x)^2 * (\cos[k\, d \cdot x])^2.$$

Should we leave this as a parameter for xmgrace to fit? Yes we can! (Bob the builder...)

fit parameter for slit separation.

So this would go into xmgrace as a 3 parameter fit.

$$y = \underline{a_0} + 4*\underline{a_1} * (\sin(k*0.04e\text{-}3*x)/(k*0.04e\text{-}3*x))^\wedge 2$$
$$*(\cos(k*\underline{a2}*x))^\wedge 2$$ with the # for k put in.

Initial guesses & bounding:

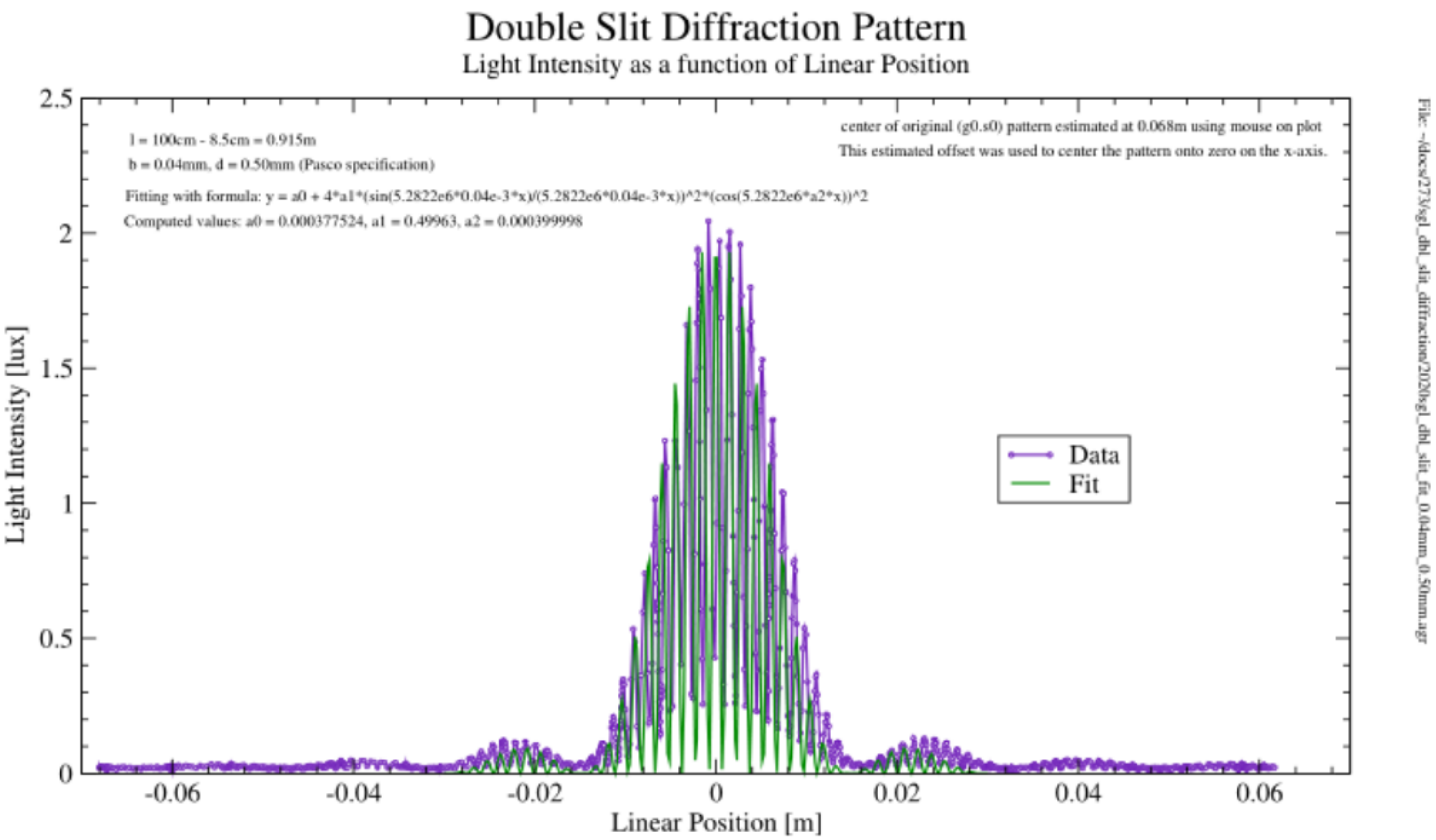
a_0 guess 0.0002 lux ← small offset if needed.
a_1 guess 0.5 lux ← pattern peak is approximately 2 lux & we included the $\underline{4}\,I_0$
a_2 guess 0.0004 m or 0.4 mm. Pasco specification for the slit separation is 0.5 mm.

On the Advanced tab:

Load: Function

Start Load at −0.03 Stop Load at 0.03 # of points: 600

This will produce a fit as shown below:



The imported data (1401 points) is also undersampled and that may be interfering with the quality of fit by xmgrace.

Some explanation of xmgrace non-linear fit statistics

Source: statisticshowto.com

Four parameters are calculated by xmgrace a) to d) below:

a) χ^2: 1) "goodness of fit test" (does the sample data match the population data?)

2) "test for independence" (does y depend on x?)

3) large χ^2: data does not fit well, no relationship
small χ^2: observed data fits expected very well, yes there is a relationship.

4) $$\chi_c^2 = \sum_i \frac{(\text{Observed}_i - \text{expected}_i)^2}{\text{expected}_i}$$

c ⇒ degrees of freedom: # of catagories − 1

b) Correlation Coefficient: usually ρ or r & between −1 and +1

c) RMS relative error: $$\left[\frac{1}{n}\sum_i^n (O_i - E_i)^2\right]^{1/2}$$

d) Theil U : U = "unbiased" + 3 more categories including Theil's U Statistic

(1) Used in Finance + (2) $U_1 \to$ forecast accuracy (1958)
$U_2 \to$ forecast quality (1966)

A = change in actual earnings
P = predicted change in earnings

$$U_1 = \left[\sum_{i=1}^{n} (P_i - A_i)^2\right]^{1/2} \Big/ \left[\sum_{i=1}^{n} A_i^2\right]^{2}$$

$$U_2 = \frac{\left[\frac{1}{n}\sum_{i=1}^{n} (A_i - P_i)^2\right]^{1/2}}{\left[\frac{1}{n}\sum_{i=1}^{n} A_i^2\right]^{1/2} + \left[\frac{1}{n}\sum_{i=1}^{n} P_i^2\right]^{1/2}}$$

Theil's original formula, some others confusingly exist.

All these describe a "goodness of fit" sort of quality, they do not put error bars on the fits results for $A_0 \ldots A_N$.

To do so requires Monte Carlo Simulation techniques whereby a small level of "noise" is introduced into the data and the fit redone. This is done many times to build up a library of fits and from there an error assigned to the fit results. This is not a trivial task, but used widely to understand the uncertainty in prediction models.

The following discussion was previously (2016 → 2019 approximately) and is not to be used + is not required if xmgrace is used to do a non-linear fit to the data. Also below are steps to convert RMS angle [radians] to linear distance X. This is not needed if "rack" is set in the RMS's output. That setting does the conversion for you. The text is being left in the lab for future consideration.

Step 2: 

Properly scale the x-axis = linear displacent. The RMS data is proportional to linear* displacement (not equal to). For a true distance of 0.182 m, the RMS linear distance will be 0.341 m, a value that is too large. Thus the x-axis must be scaled down by 0.182 m/0.341 m:

① Create a new set (gØ.s2) where
 X = gØ.s1.x * 0.182/0.341
 Y = gØ.s1.y

② Hide the old .s1 set and scale + display the new set

③ Label the new set : scaled and centered X.

Step 3: Remove zero x values if present
(if using the data set to generate a plot, not a fit)

The calculation of the intensity pattern requires dividing by $\beta = \frac{\pi a}{\lambda} \sin\theta$. If β goes to zero, the software can't plot $I_0 (\sin\beta)^2/\beta^2$ due to a divide by zero condition. This happens if $x = 0$.

① Duplicate the "scaled + centered" set. This is done by selecting the set, right clicking on it, Duplicate.

② Bring up the newly created set and delete row(s) near the middle of the set where X = Ø.

③ Label this set "nozeros".

Step 4: Calculate $\beta = \frac{\pi a}{\lambda} \sin\theta$

① Determine what $\sin\theta$ is in terms of x + l

② Calculate the value of $\pi a/\lambda$ by calculator; watch units cancel.

③ Create a new set by calculation for $\beta = \frac{\pi a}{\lambda} f(x, l)$ where the function is your equivalent to $\sin\theta$.
④ Label this set "Beta"

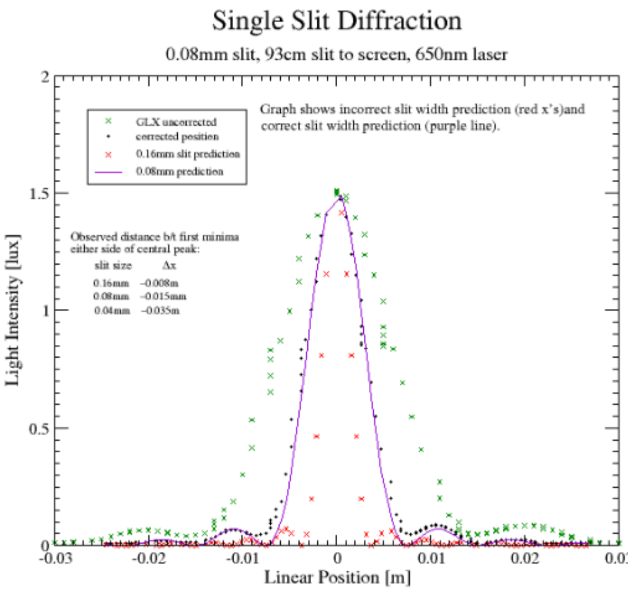
Step 5 Calculate $I_0 \frac{\sin^2\beta}{\beta^2} = I(x)$

① Obtain your value for I_0 which is the maximum intensity value observed. It will be near $X = \emptyset$. Look @ the old set that still has the $X = \emptyset$ data.
② Calculate a new set. For example if I_0 is 3.48 lux and g0.55 has the β values in the Y column, the new set would have:

X= g0.s5.x
Y= 3.48*(sin(g0.s5.y))^2/g0.s5.y^2

At this point you hopefully have a predicted single slit pattern line plotted which agrees very well with the data collected. Present the data as circles and the theoretical line as a line or dashed line in xmgrace. See below:

Some extra information is presented here. It was collected to insure the slit width and Δx values observed + predicted matched well. For the report just the corrected data + the prediction line need to appear.



Data using other values of slit width shown

Hints and Help

Turn backlight on: Home, Settings, ↓ down to backlight, ✓, choose ON.

Calculation of $\frac{\pi a}{\lambda} = \frac{3.14159 * 0.16\,mm}{650\,nm} = 773.32$ & $l = 93\,cm = 0.93\,m$

Assuming s3
Calculation of β: $773.32 * \frac{g0.s3.x}{\sqrt{(g0.s3.x)^2 + 0.93^2}}$

(which is: $\frac{\pi a}{\lambda} \cdot \underbrace{\frac{x}{\sqrt{x^2+l^2}}}_{\sin\theta}$)

X = g0.s3.x
Y = β = 773.32 * g0.s3.x / sqrt((g0.s3.x)^2 + 0.93^2))

For other a values:

a	$\pi a/\lambda$
0.16 mm	773.32
0.08 mm	386.66
0.04 mm	193.33

Calculation for $I(x) = I_0 \frac{\sin^2\beta}{\beta^2}$

$I_x = I_0$ (sin(g0.s4.y))^2/g0.s4.y^2
where:
I_0 has been chosen by looking through the data table to find the max value.

It is very useful to label your sets in xmgrace. For the top figure on page 9, the following sets were present in xmgrace, just as an example:

2020: create the ✓ sets, do not create the others.

✓ − G0.S0 [2][149] "original data"
✓ + G0.S1 [2][149] "data centered @ zero" (moved data; peak I @ 0m)
✗ − G0.S2 [2][149] "scaled and centered X" (corrected for scale $\frac{0.182}{0.341}$)
✗ + G0.S3 [2][144] "no zeros, copy of G0.S2" (removed X=0 data points)
✗ − G0.S4 [2][144] "0.16mm Beta calculation" ($\beta = \frac{\pi a}{\lambda}\sin\theta$ calc)
✗ + G0.S5 [2][144] "0.16mm prediction" (predicted curve a=0.16mm)
✗ − G0.S6 [2][144] "0.08mm Beta calculation" ($\beta = \frac{\pi a}{\lambda}\sin\theta$ calc)
✗ + G0.S7 [2][144] "0.08mm prediction" (predicted curve a=0.08mm)

shown or hidden

Radians: If the GLX data was not exported in meters of linear position, it is probably in radians. The metric is actually the RMS pulley's position in rad, not the angle of interest θ.

2023: we chose "rack" for the RMS sensor: rotational θ to linear x