# Blackbody Radiation

## Introduction:

This is one of the key experiments that led to the development of Quantum Mechanics.

The spectrum of an incandescent light bulb is scanned by hand using a prism spectrophotometer that measures relative light intensity as a function of angle. A Broad Spectrum Light Sensor is used with a prism so the entire spectrum from approximately 400 nm to 2500 nm can be scanned without th overlapping orders caused by a grating. The wavelengths corresponding to the angles are calculated using the equations for a prism spectrophotometer. The relative light intensity can then be plotted as a function of wavelength as the spectrum is scanned, resulting in the characteristic blackbody curve. The intensity of the light bulb is increased, increasing the temperature, and the scan is repeated to show how the curves nest with a shift in the peak wavelength.

The temperature of the filament of the bulb can be estimated indirectly by determining the resistance of the bulb from the measured voltage and current. From the temperature, the theoretical peak wavelength can be calculated and compared to the measured peak wavelength.

This experiment should be performed in a room with reduced light levels although complete darkness is not required.

*Written by Chuck Hunt*

## Equipment:

| | | |
|---|---|---|
| | **INCLUDED:** | |
| 1 | Prism Spectrophotometer Kit | OS-8544 |
| 1 | Optics Bench (60 cm) | OS-8541 |
| 1 | Spectrophotometer Accessory Kit | OS-8537 |
| 1 | Aperture Bracket | OS-8534B |
| 1 | Broad Spectrum Light Sensor | PS-2150 |
| 1 | Rotary Motion Sensor | PS-2120A |
| 1 | Voltage Sensor | UI-5100 |
| 1 | Replacement Bulb (10 pk) | SE-8509 |
| 1 | Banana Plug Cord-Black (5 pack) | SE-9751 |
| **NOT INCLUDED, BUT REQUIRED:** | | |
| ~~1~~ | 850 Universal Interface | UI-5000 |
| ~~1~~ | PASCO Capstone | |

* 850 interface not available in dept for 2018.

* We will use GLX's + export data to pull into xmgrace for plotting.

① Spectrometer set up: See p16 + p17 then p3,4

② We must know 0° is really 0° for light's path from source to sensor.

## Theory:

The intensity per wavelength, $I_\lambda(\lambda,T)$, as a function of wavelength of radiation emitted by an ideal body (a blackbody since a ideal emitter must also be an ideal absorber) is given by Planck's Radiation Law:

$$I_\lambda(\lambda,T)=\frac{2\pi c^2 h}{\lambda^5}\left(\frac{1}{e^{(hc/\lambda kT)}-1}\right) \qquad (1)$$

where c is the speed of light in a vacuum, h is Planck's constant, k is Boltzmann's constant, T is the absolute temperature of the body, and λ is the wavelength of the radiation. Any real object must emit less at all wavelengths.

The wavelength with the greatest intensity is given by

$$\lambda_{max} = (\text{constant})/T = (0.002898\ \text{m*K})/T \qquad (2)$$

The temperature of the blackbody light filament can be calculated using the resistance of the filament while it is lit. We find the resistance (R) by measuring the voltage (V) and the current (I) and using R = V/I. The resistance of the tungsten filame is a nonlinear function of the temperature. Using the measured resistance to calculate the temperature is discussed in Appendix ~~1.~~ 2

The wavelength is determined by measuring the angle at which the light is dispersed by a prism. The relationship between the angle and the wavelength is discussed in Appendix ~~2.~~ 1

Appendix #'s incorrect

$\lambda_{max} \cdot T = 2.898e\text{-}3\ m \cdot K$

"red hot" → 900-1000 K

tungsten filament ~ 2k-3k K

or 2550°C ~

75W to 100W

Energy per unit volume per wavelength $S_\lambda \left[\frac{J}{m^3 \cdot m}\right]$

$$S_\lambda = \frac{8\pi h c}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$$

$$= A_0 \cdot \frac{1}{\lambda^5} \cdot \frac{1}{e^{A_1/\lambda} - 1}$$

$1W = 1J/s$

Radiation Power Density $\left[W/m^3\right]$

$$S(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

To find via fit, value of T:

$A_0 = 8\pi h c$

$A_1 = \frac{hc}{k} \cdot \frac{1}{T} = \#$

fit yields $A_0$ & $A_1$

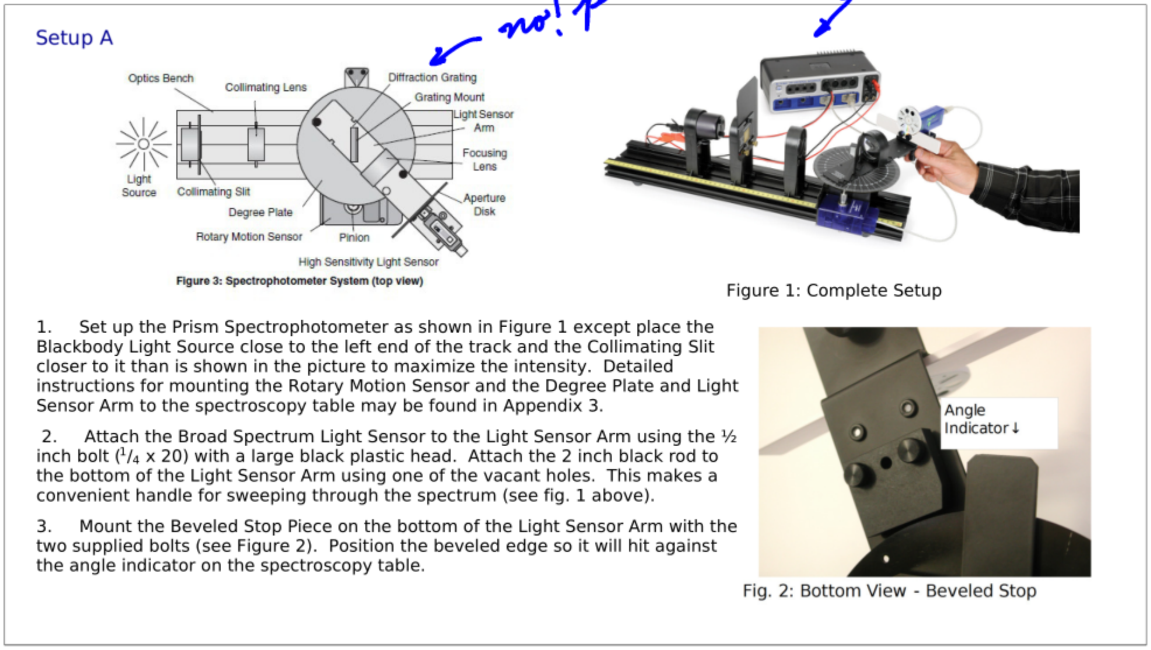
$A_1$ gives value of T.

$$= \frac{A_0}{(g\# \cdot s\# \cdot x)\wedge 5} \cdot \frac{1}{\underset{\uparrow}{e}^{A_1/\lambda} - 1} + A_2$$

2.71828

850 → G2X as 850
not available.

no! prism



Setup A

**Figure 3: Spectrophotometer System (top view)**

Figure 1: Complete Setup

1. Set up the Prism Spectrophotometer as shown in Figure 1 except place the Blackbody Light Source close to the left end of the track and the Collimating Slit closer to it than is shown in the picture to maximize the intensity. Detailed instructions for mounting the Rotary Motion Sensor and the Degree Plate and Light Sensor Arm to the spectroscopy table may be found in Appendix 3.

2. Attach the Broad Spectrum Light Sensor to the Light Sensor Arm using the ½ inch bolt ($^1/_4$ x 20) with a large black plastic head. Attach the 2 inch black rod to the bottom of the Light Sensor Arm using one of the vacant holes. This makes a convenient handle for sweeping through the spectrum (see fig. 1 above).

3. Mount the Beveled Stop Piece on the bottom of the Light Sensor Arm with the two supplied bolts (see Figure 2). Position the beveled edge so it will hit against the angle indicator on the spectroscopy table.

Fig. 2: Bottom View - Beveled Stop

Light Source: may replace with desk lamp
* BB light source bulbs easily burned out
× " " " is dim.

More experimentation is needed with this source issue.

Energy per unit volume per wavelength $S_\lambda \left[\frac{J}{m^3 \cdot m}\right]$

① $$S_\lambda = \frac{8\pi h c}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$$ } energy density per $\lambda$

Radiation Power Density $\left[W/m^3\right]$ and $1\,W = 1\,\frac{\text{Joule}}{\text{second}}$

② $$S(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$ } power density per $\lambda$

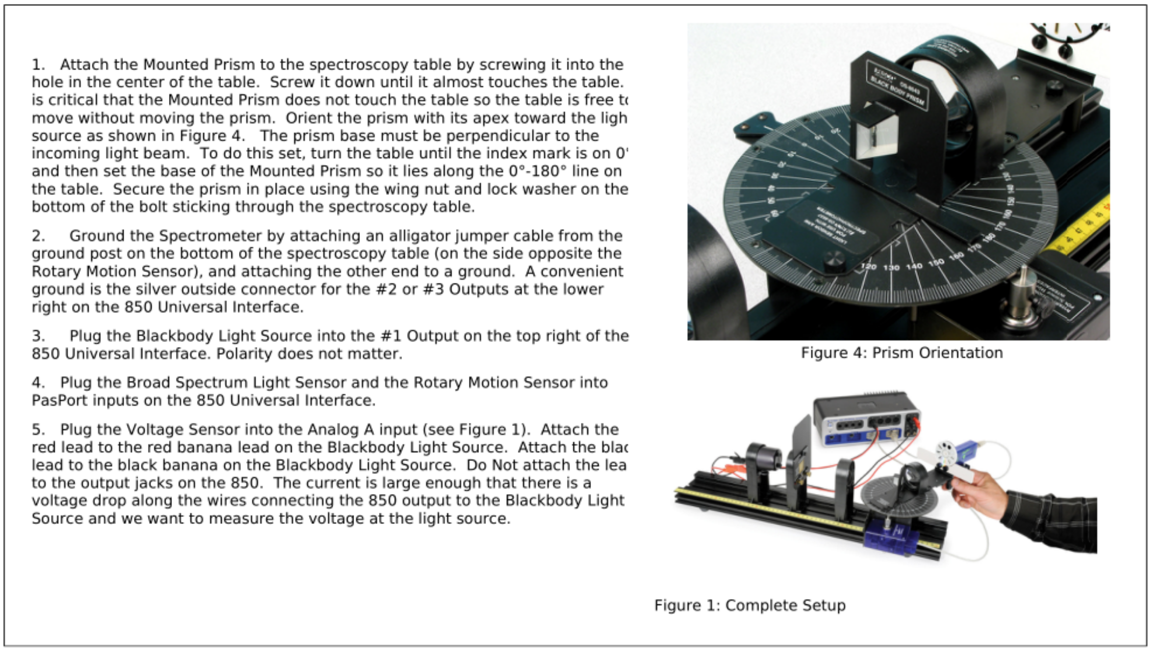
From hyperphysics online: To find the radiated power per unit area from a surface at this temperature, multiply the energy density by c/4. Here are links explaining the c/4 factor extensively.

S is used for both formulas above → caution needed ¡

$2\pi c^2 h = 2\pi \cdot (2.9979e8)^2 \cdot 6.6261e\text{-}34 = 374.17e\text{-}18\ m^4 kg/s^3$ fitting to ②

$hc = 1.98645e\text{-}25\, m^3 kg/s^2$ or $198.64e\text{-}27\ m^3 kg/s^2$

$hc/k = hc \div 1.38065e\text{-}23\, \frac{m^2 kg}{s^2 K} = 14.388e\text{-}3\ m \cdot K$



1. Attach the Mounted Prism to the spectroscopy table by screwing it into the hole in the center of the table. Screw it down until it almost touches the table. is critical that the Mounted Prism does not touch the table so the table is free t move without moving the prism. Orient the prism with its apex toward the ligh source as shown in Figure 4. The prism base must be perpendicular to the incoming light beam. To do this set, turn the table until the index mark is on 0 and then set the base of the Mounted Prism so it lies along the 0°-180° line on the table. Secure the prism in place using the wing nut and lock washer on the bottom of the bolt sticking through the spectroscopy table.

2. Ground the Spectrometer by attaching an alligator jumper cable from the ground post on the bottom of the spectroscopy table (on the side opposite the Rotary Motion Sensor), and attaching the other end to a ground. A convenient ground is the silver outside connector for the #2 or #3 Outputs at the lower right on the 850 Universal Interface.

3. Plug the Blackbody Light Source into the #1 Output on the top right of the 850 Universal Interface. Polarity does not matter.

4. Plug the Broad Spectrum Light Sensor and the Rotary Motion Sensor into PasPort inputs on the 850 Universal Interface.

5. Plug the Voltage Sensor into the Analog A input (see Figure 1). Attach the red lead to the red banana lead on the Blackbody Light Source. Attach the blac lead to the black banana on the Blackbody Light Source. Do Not attach the lea to the output jacks on the 850. The current is large enough that there is a voltage drop along the wires connecting the 850 output to the Blackbody Light Source and we want to measure the voltage at the light source.

Figure 4: Prism Orientation

Figure 1: Complete Setup

## METHOD

NB: Sample rate for both sensors (RMS + broad spectrum light sensor) must be set to:

$$\boxed{40 \text{ samples/sec}}$$

Use HOME, Sensors, ✓ + change sample rate to the above value.

## ANALYSIS

The data collected may be offset vertically. Thus the equation to use in xmgrace should also include a parameter for this offset. This can be entered into the nonlinear fit dialog as:

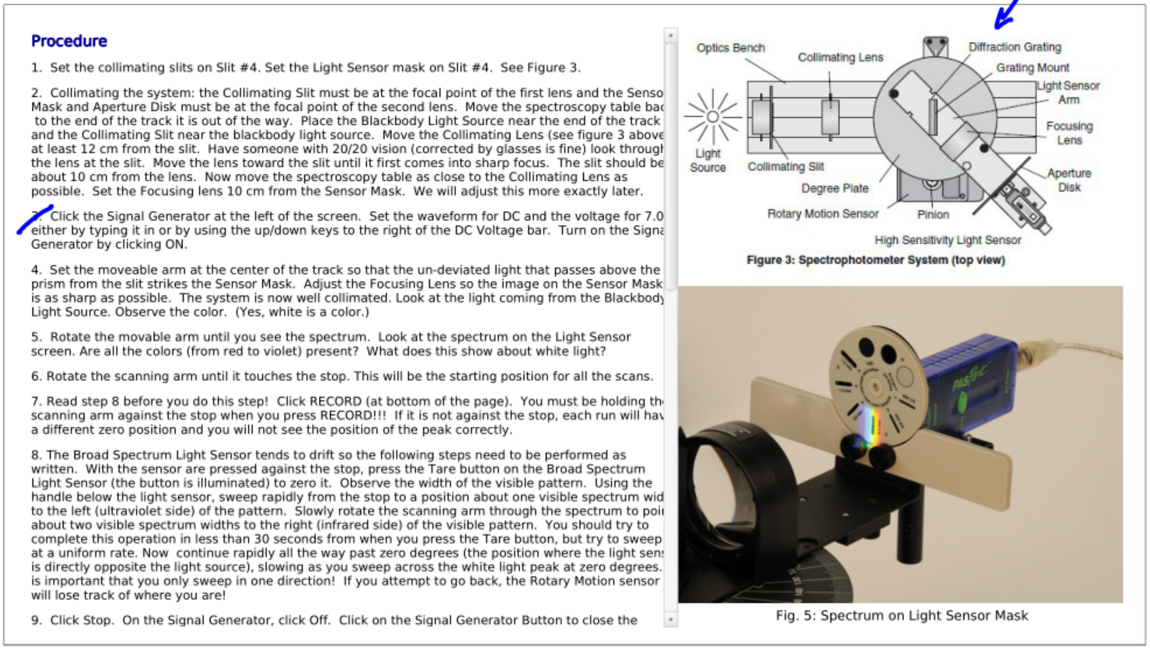
$$y = A_0 + A_1/x \wedge 5 * (1/(2.71828 \wedge (A_2/x) - 1))$$

With a value for $A_2$ from the fit, T[K] can be found.

A second value for T can be found by simply using the peak's value of $\lambda = \lambda_{max}$ and the eq'n:

$$\lambda_{max} T = \text{constant}.$$

no! prism



## Procedure

1. Set the collimating slits on Slit #4. Set the Light Sensor mask on Slit #4. See Figure 3.

2. Collimating the system: the Collimating Slit must be at the focal point of the first lens and the Senso Mask and Aperture Disk must be at the focal point of the second lens. Move the spectroscopy table bac to the end of the track it is out of the way. Place the Blackbody Light Source near the end of the track and the Collimating Slit near the blackbody light source. Move the Collimating Lens (see figure 3 above at least 12 cm from the slit. Have someone with 20/20 vision (corrected by glasses is fine) look throug the lens at the slit. Move the lens toward the slit until it first comes into sharp focus. The slit should be about 10 cm from the lens. Now move the spectroscopy table as close to the Collimating Lens as possible. Set the Focusing lens 10 cm from the Sensor Mask. We will adjust this more exactly later.

3. Click the Signal Generator at the left of the screen. Set the waveform for DC and the voltage for 7.0 either by typing it in or by using the up/down keys to the right of the DC Voltage bar. Turn on the Signa Generator by clicking ON.

4. Set the moveable arm at the center of the track so that the un-deviated light that passes above the prism from the slit strikes the Sensor Mask. Adjust the Focusing Lens so the image on the Sensor Mask is as sharp as possible. The system is now well collimated. Look at the light coming from the Blackbody Light Source. Observe the color. (Yes, white is a color.)

5. Rotate the movable arm until you see the spectrum. Look at the spectrum on the Light Sensor screen. Are all the colors (from red to violet) present? What does this show about white light?

6. Rotate the scanning arm until it touches the stop. This will be the starting position for all the scans.

7. Read step 8 before you do this step! Click RECORD (at bottom of the page). You must be holding th scanning arm against the stop when you press RECORD!!! If it is not against the stop, each run will hav a different zero position and you will not see the position of the peak correctly.

drift
⇒30 sec
run

8. The Broad Spectrum Light Sensor tends to drift so the following steps need to be performed as written. With the sensor are pressed against the stop, press the Tare button on the Broad Spectrum Light Sensor (the button is illuminated) to zero it. Observe the width of the visible pattern. Using the handle below the light sensor, sweep rapidly from the stop to a position about one visible spectrum wid to the left (ultraviolet side) of the pattern. Slowly rotate the scanning arm through the spectrum to poi about two visible spectrum widths to the right (infrared side) of the visible pattern. You should try to complete this operation in less than 30 seconds from when you press the Tare button, but try to sweep at a uniform rate. Now continue rapidly all the way past zero degrees (the position where the light sen is directly opposite the light source), slowing as you sweep across the white light peak at zero degrees. is important that you only sweep in one direction! If you attempt to go back, the Rotary Motion sensor will lose track of where you are!

9. Click Stop. On the Signal Generator, click Off. Click on the Signal Generator Button to close the

Figure 3: Spectrophotometer System (top view)

Fig. 5: Spectrum on Light Sensor Mask

A demonstration of a run will be provided.

## ANALYSIS

2018: Your data may be shifted horizontally for an unknown reason. Consult the handout showing the four graphs. The RMS [radians] for you peak % Intensity should be very similar. If that peak is shifted → or ← too much you may not be able to get the conversion to Table angle to work. This is a result of the limited range over which a prism functions (in λ) and the limited range that equation A5 can be used. You can try shifting your data in xmgrace by creating a new set adding/subtracting the approximate amount needed. Consult your lab instructor.

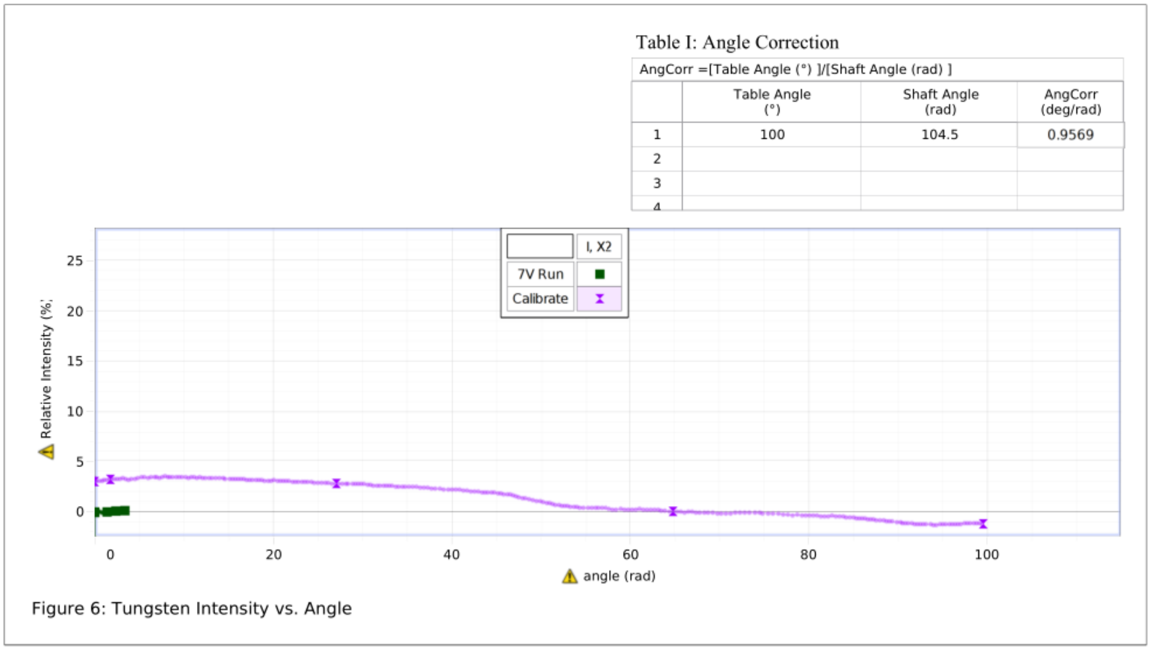


Table I: Angle Correction

AngCorr =[Table Angle (°) ]/[Shaft Angle (rad) ]

| | Table Angle (°) | Shaft Angle (rad) | AngCorr (deg/rad) |
|---|---|---|---|
| 1 | 100 | 104.5 | 0.9569 |
| 2 | | | |
| 3 | | | |
| 4 | | | |

Figure 6: Tungsten Intensity vs. Angle

Angle correction:

We need the ability to convert RMS angle to table angle since table angle is the θ used in n(sinθ) formula. To accomplish this.

① Put GLX in digits display mode (available on HOME screen).

② Display RMS angle to 3 or 4 digits.

③ Start at table angle zero + press record/run

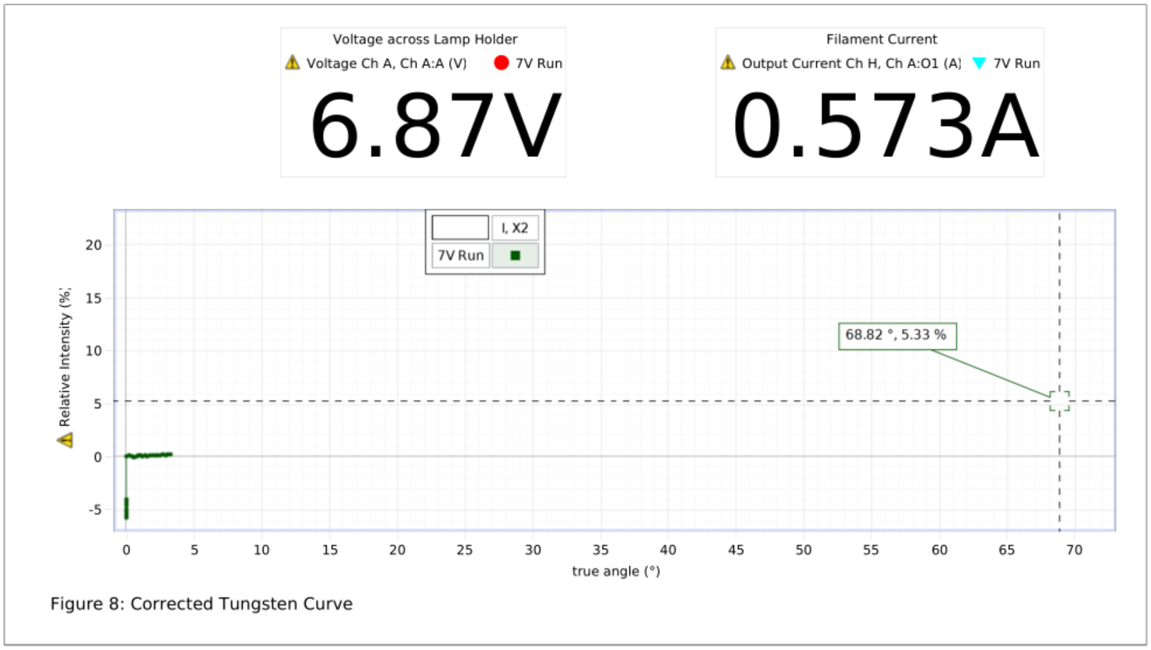
④ Record RMS angle. Increment table angle by 10° + record RMS angle. Keep going until physical stop is reached.

RMS angle = X   Table angle = Y

⑤ Plot Table Angle as a function of RMS Angle + do a linear fit in Xmgrace. The equation of fit can then be used to create a new set of intensity as a function of table angle.

Optional for 2018:

Below are details of determining filament temperature by an independent method.



Voltage across Lamp Holder

⚠ Voltage Ch A, Ch A:A (V) ● 7V Run

6.87V

Filament Current

⚠ Output Current Ch H, Ch A:O1 (A) ▼ 7V Run

0.573A

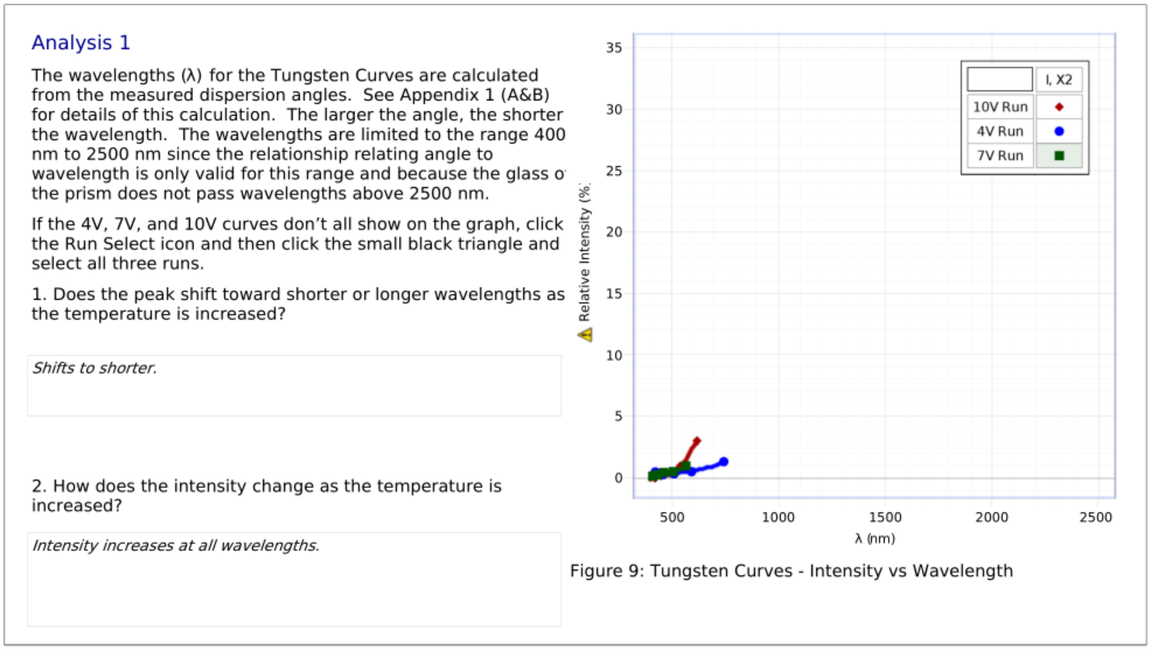
Figure 8: Corrected Tungsten Curve

- * This will have to be done separately using Fluke meters: Voltage + current measurements!
- * Use the variac + start lamp @ max brightness ~130 $V_{rms}$
- * Do four voltages:

130V
110V
85V
60V
} start angle sweeps @ physical stop + continue past max intensity: blue → red ✓

NB: keep track of Run# and voltage

- * Use the GLX file manage to copy the file to a USB stick.
- * Then export each run to USB stick. Use a nearly empty stick.
  - This is done in the Table display on the GLX.
  - Select the Run to be displayed in the table, press F4, 8, ↓ down arrow, ✓ check to change UniCode to ASCII, F1 = ok.



Analysis 1

The wavelengths (λ) for the Tungsten Curves are calculated from the measured dispersion angles. See Appendix 1 (A&B) for details of this calculation. The larger the angle, the shorter the wavelength. The wavelengths are limited to the range 400 nm to 2500 nm since the relationship relating angle to wavelength is only valid for this range and because the glass o the prism does not pass wavelengths above 2500 nm.

If the 4V, 7V, and 10V curves don't all show on the graph, click the Run Select icon and then click the small black triangle and select all three runs.

1. Does the peak shift toward shorter or longer wavelengths as the temperature is increased?

*Shifts to shorter.*

2. How does the intensity change as the temperature is increased?

*Intensity increases at all wavelengths.*

Figure 9: Tungsten Curves - Intensity vs Wavelength

## ANALYSIS

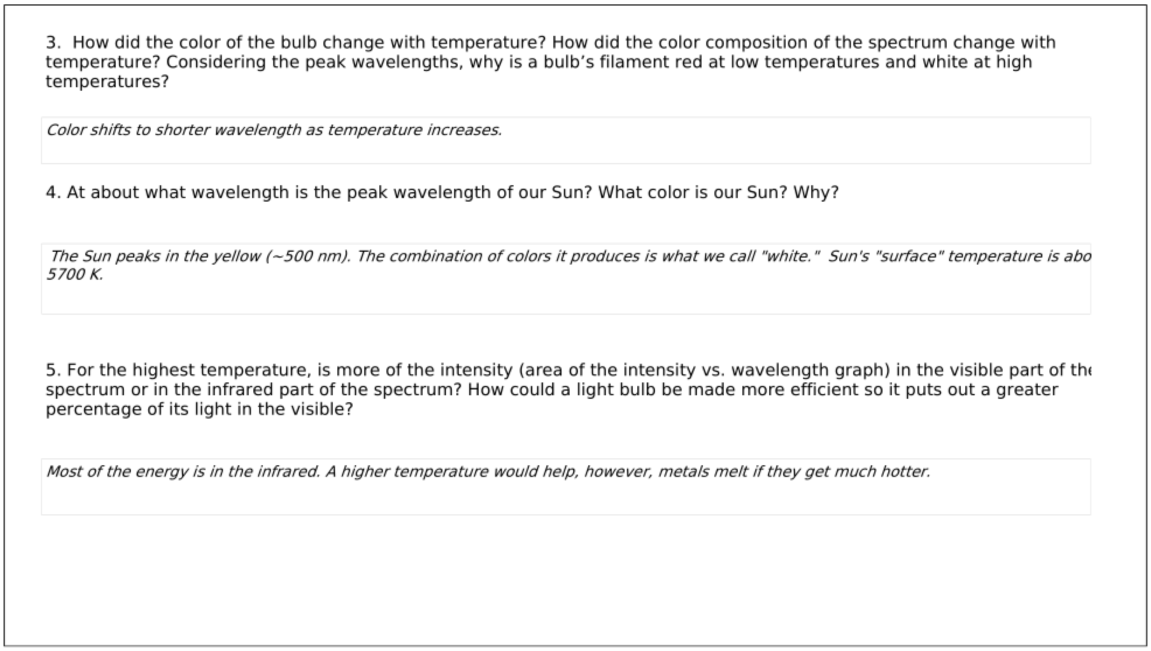
① Import data in xmgrace as block data with X from Column 2 and Y from Column 3. Column 1 in the txt file will be the sample # (integer) and is not used.

② This will give a plot of % Intensity as a function of RMS angle.

③ Convert to RMS angle to Table Angle using the fit eq'n from previous steps.

④ Convert Table angle to λ [nm] using the equations in Appendix 1.

⑤ In xmgrace, fit one plot of %I vs. λ[nm] to get a value for the filaments' temperature



3. How did the color of the bulb change with temperature? How did the color composition of the spectrum change with temperature? Considering the peak wavelengths, why is a bulb's filament red at low temperatures and white at high temperatures?

*Color shifts to shorter wavelength as temperature increases.*

4. At about what wavelength is the peak wavelength of our Sun? What color is our Sun? Why?

*The Sun peaks in the yellow (~500 nm). The combination of colors it produces is what we call "white." Sun's "surface" temperature is abo 5700 K.*

5. For the highest temperature, is more of the intensity (area of the intensity vs. wavelength graph) in the visible part of th spectrum or in the infrared part of the spectrum? How could a light bulb be made more efficient so it puts out a greater percentage of its light in the visible?

*Most of the energy is in the infrared. A higher temperature would help, however, metals melt if they get much hotter.*

We'll capture solar spectrum using the Ocean Optics spectrometers + compare to synthetic spectra.

Once BB curves are imported into grace:

Analysis: 1) Identify peak λ using the mouse cursor in xmgrace.

2) Hopefully peak λ is shifting left for hotter bulb runs ...

3) Use Wein's displacement law to calculate $\lambda_{max}$ + compare to data plots.

* 4) Showing a fit using xmgrace to one run, giving values of $A_0$, $A_1$, $A_2$.

Below:

Filament temperature via voltage + current values. Not required for 2018.

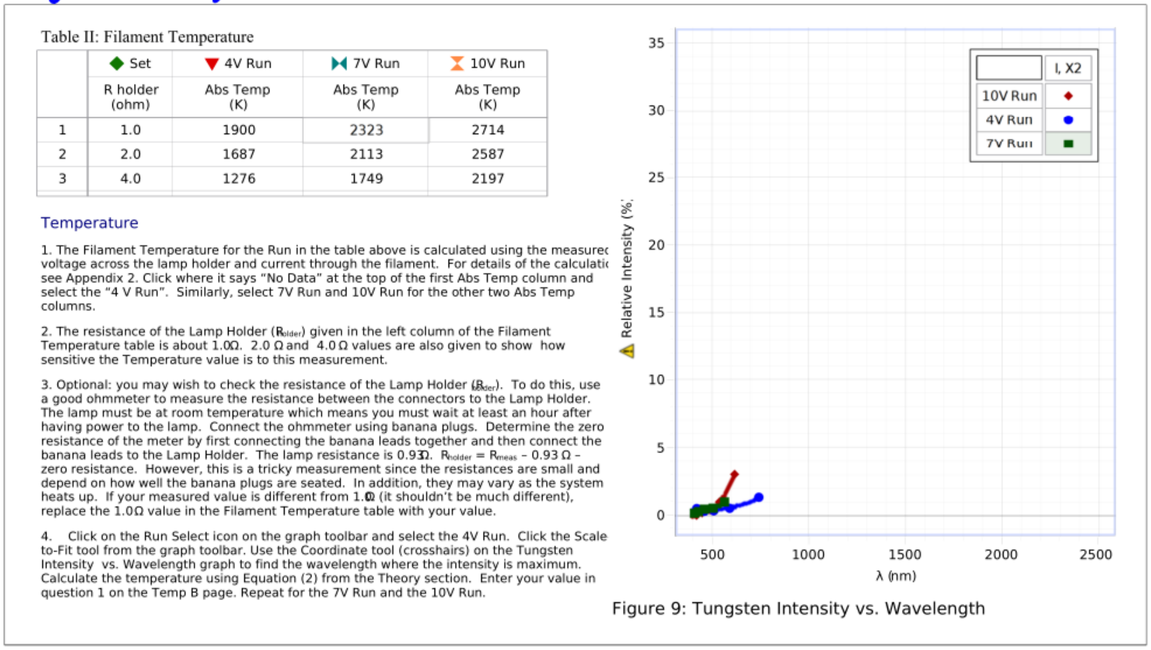


Table II: Filament Temperature

|  | ◆ Set | ▼ 4V Run | ⧓ 7V Run | ⧗ 10V Run |
|---|---|---|---|---|
|  | R holder (ohm) | Abs Temp (K) | Abs Temp (K) | Abs Temp (K) |
| 1 | 1.0 | 1900 | 2323 | 2714 |
| 2 | 2.0 | 1687 | 2113 | 2587 |
| 3 | 4.0 | 1276 | 1749 | 2197 |

Temperature

1. The Filament Temperature for the Run in the table above is calculated using the measured voltage across the lamp holder and current through the filament. For details of the calculati see Appendix 2. Click where it says "No Data" at the top of the first Abs Temp column and select the "4 V Run". Similarly, select 7V Run and 10V Run for the other two Abs Temp columns.

2. The resistance of the Lamp Holder ($R_{holder}$) given in the left column of the Filament Temperature table is about 1.0Ω. 2.0 Ω and 4.0 Ω values are also given to show how sensitive the Temperature value is to this measurement.

3. Optional: you may wish to check the resistance of the Lamp Holder ($R_{holder}$). To do this, use a good ohmmeter to measure the resistance between the connectors to the Lamp Holder. The lamp must be at room temperature which means you must wait at least an hour after having power to the lamp. Connect the ohmmeter using banana plugs. Determine the zero resistance of the meter by first connecting the banana leads together and then connect the banana leads to the Lamp Holder. The lamp resistance is 0.93Ω. $R_{holder} = R_{meas} - 0.93\,\Omega -$ zero resistance. However, this is a tricky measurement since the resistances are small and depend on how well the banana plugs are seated. In addition, they may vary as the system heats up. If your measured value is different from 1.0Ω (it shouldn't be much different), replace the 1.0Ω value in the Filament Temperature table with your value.

4. Click on the Run Select icon on the graph toolbar and select the 4V Run. Click the Scale to-Fit tool from the graph toolbar. Use the Coordinate tool (crosshairs) on the Tungsten Intensity vs. Wavelength graph to find the wavelength where the intensity is maximum. Calculate the temperature using Equation (2) from the Theory section. Enter your value in question 1 on the Temp B page. Repeat for the 7V Run and the 10V Run.

Figure 9: Tungsten Intensity vs. Wavelength

Copy of Appendix 1: This step not required in 2018.

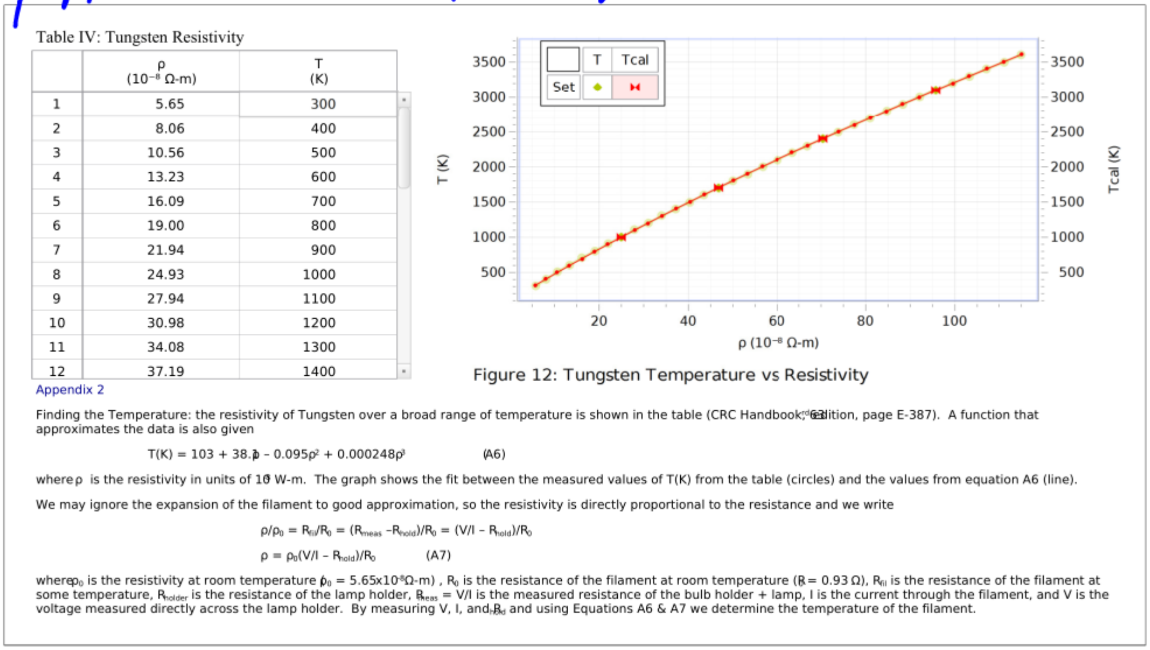


Table IV: Tungsten Resistivity

|  | ρ ($10^{-8}$ Ω-m) | T (K) |
|---|---|---|
| 1 | 5.65 | 300 |
| 2 | 8.06 | 400 |
| 3 | 10.56 | 500 |
| 4 | 13.23 | 600 |
| 5 | 16.09 | 700 |
| 6 | 19.00 | 800 |
| 7 | 21.94 | 900 |
| 8 | 24.93 | 1000 |
| 9 | 27.94 | 1100 |
| 10 | 30.98 | 1200 |
| 11 | 34.08 | 1300 |
| 12 | 37.19 | 1400 |

Figure 12: Tungsten Temperature vs Resistivity

Appendix 2

Finding the Temperature: the resistivity of Tungsten over a broad range of temperature is shown in the table (CRC Handbook, 63rd edition, page E-387). A function that approximates the data is also given

$$T(K) = 103 + 38.1\rho - 0.095\rho^2 + 0.000248\rho^3 \qquad (A6)$$

where ρ is the resistivity in units of $10^{-8}$ W-m. The graph shows the fit between the measured values of T(K) from the table (circles) and the values from equation A6 (line).

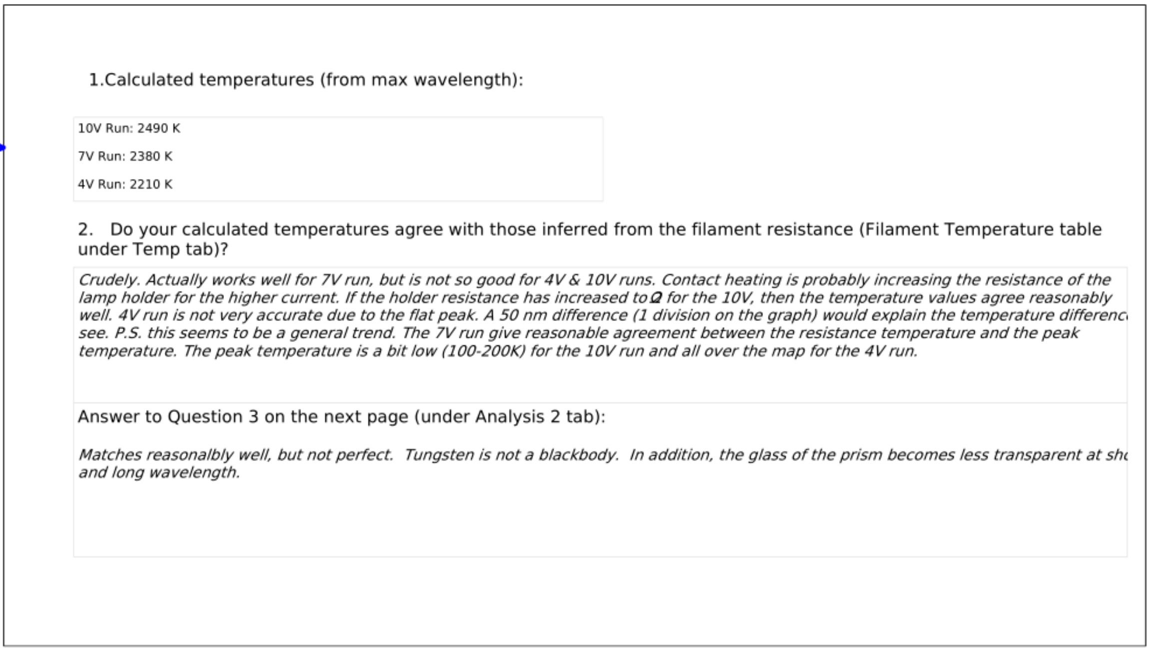
We may ignore the expansion of the filament to good approximation, so the resistivity is directly proportional to the resistance and we write

$$\rho/\rho_0 = R_{fil}/R_0 = (R_{meas} - R_{hold})/R_0 = (V/I - R_{hold})/R_0$$

$$\rho = \rho_0(V/I - R_{hold})/R_0 \qquad (A7)$$

where $\rho_0$ is the resistivity at room temperature ($\rho_0 = 5.65\times10^{-8}$ Ω-m) , $R_0$ is the resistance of the filament at room temperature ($R_0 = 0.93\,\Omega$), $R_{fil}$ is the resistance of the filament at some temperature, $R_{holder}$ is the resistance of the lamp holder, $R_{meas} = V/I$ is the measured resistance of the bulb holder + lamp, I is the current through the filament, and V is the voltage measured directly across the lamp holder. By measuring V, I, and $R_{hold}$ and using Equations A6 & A7 we determine the temperature of the filament.

2018: not required



1.Calculated temperatures (from max wavelength):

10V Run: 2490 K
7V Run: 2380 K
4V Run: 2210 K

2. Do your calculated temperatures agree with those inferred from the filament resistance (Filament Temperature table under Temp tab)?

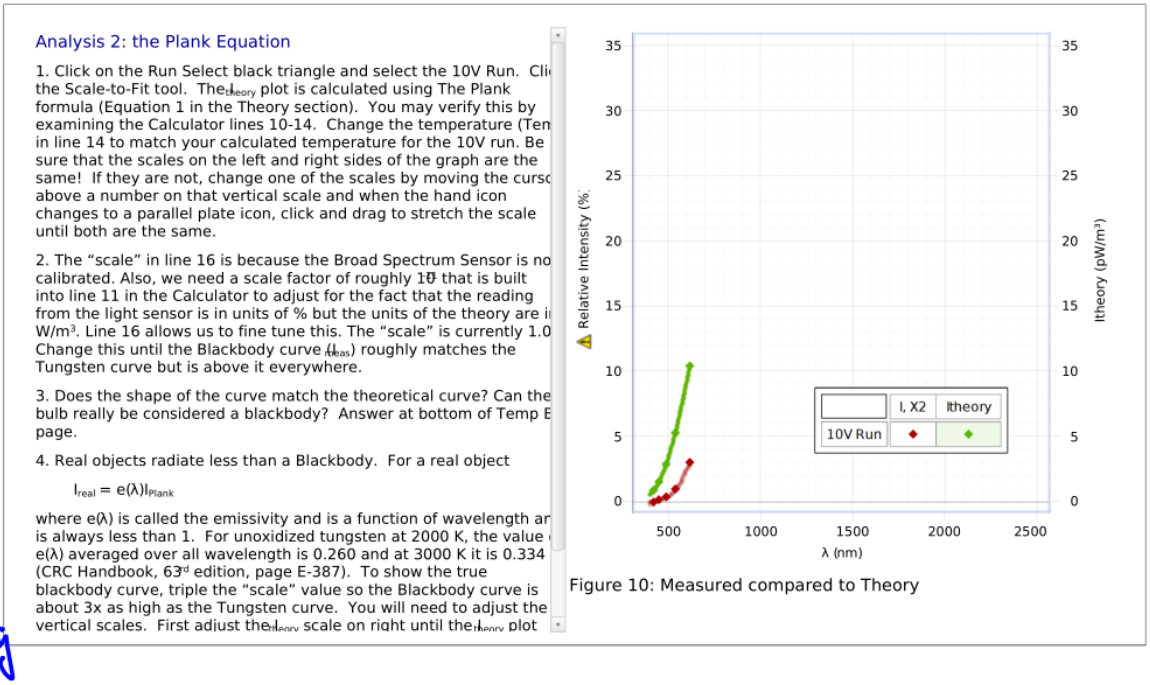
*Crudely. Actually works well for 7V run, but is not so good for 4V & 10V runs. Contact heating is probably increasing the resistance of the lamp holder for the higher current. If the holder resistance has increased to 2Ω for the 10V, then the temperature values agree reasonably well. 4V run is not very accurate due to the flat peak. A 50 nm difference (1 division on the graph) would explain the temperature differenc see. P.S. this seems to be a general trend. The 7V run give reasonable agreement between the resistance temperature and the peak temperature. The peak temperature is a bit low (100-200K) for the 10V run and all over the map for the 4V run.*

Answer to Question 3 on the next page (under Analysis 2 tab):

*Matches reasonalbly well, but not perfect. Tungsten is not a blackbody. In addition, the glass of the prism becomes less transparent at sho and long wavelength.*

Below:

Not apropos as we can fit in xmgrace & use the complete curve.



## Analysis 2: the Plank Equation

1. Click on the Run Select black triangle and select the 10V Run. Cli the Scale-to-Fit tool. The $I_{theory}$ plot is calculated using The Plank formula (Equation 1 in the Theory section). You may verify this by examining the Calculator lines 10-14. Change the temperature (Tem in line 14 to match your calculated temperature for the 10V run. Be sure that the scales on the left and right sides of the graph are the same! If they are not, change one of the scales by moving the curso above a number on that vertical scale and when the hand icon changes to a parallel plate icon, click and drag to stretch the scale until both are the same.

2. The “scale” in line 16 is because the Broad Spectrum Sensor is no calibrated. Also, we need a scale factor of roughly 10 that is built into line 11 in the Calculator to adjust for the fact that the reading from the light sensor is in units of % but the units of the theory are i $W/m^3$. Line 16 allows us to fine tune this. The “scale” is currently 1.0 Change this until the Blackbody curve ($I_{meas}$) roughly matches the Tungsten curve but is above it everywhere.

3. Does the shape of the curve match the theoretical curve? Can the bulb really be considered a blackbody? Answer at bottom of Temp E page.

4. Real objects radiate less than a Blackbody. For a real object

$$I_{real} = e(\lambda)I_{Plank}$$

where e(λ) is called the emissivity and is a function of wavelength ar is always less than 1. For unoxidized tungsten at 2000 K, the value e(λ) averaged over all wavelength is 0.260 and at 3000 K it is 0.334 (CRC Handbook, 63rd edition, page E-387). To show the true blackbody curve, triple the “scale” value so the Blackbody curve is about 3x as high as the Tungsten curve. You will need to adjust the vertical scales. First adjust the $I_{theory}$ scale on right until the $I_{theory}$ plot

3× due to emissivity

|  | I, X2 | Itheory |
|---|---|---|
| 10V Run | ◆ | ◆ |

Figure 10: Measured compared to Theory

## Appendix 1: Finding the wavelength as a function of angle.

The index of refraction of the prism glass varies with the wavelength of the light. To determine the wavelength as a function of the angle, the relationship between the index of refraction and the angle is determined using Snell's Law at each face of the prisı and some geometry and basic trigonometry.

$\sin 60^0 = n \sin\theta_2$ (A1) and $\sin\theta = n\sin\theta_3$ (A2) ← **Snell's**

where n is the index of refraction of the prism.

$n\sin\theta_3 = n\sin(60^0 - \theta_2) = n(\sin 60^0\cos\theta_2 - \cos 60^0\sin\theta_2)$

$= n\sin 60^0\cos\theta_2 - \cos 60^0\sin 60^0$ (using Equation A1)

Rearranging this and using Equation A2 yields

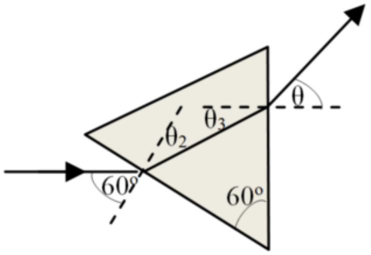
$n\cos\theta_2 = (\sin\theta/\sin 60^0) + \sin 60^0$ (A3)

Squaring Equations A1 and A3 and adding them to gives

$n^2(\sin^2\theta_2 + \cos^2\theta_2) = n^2 = [(\sin\theta/\sin 60^0) + \sin 60^0]^2 + \sin^2 60^0$

Putting in values for sin 60⁰ and cos 60⁰ yields

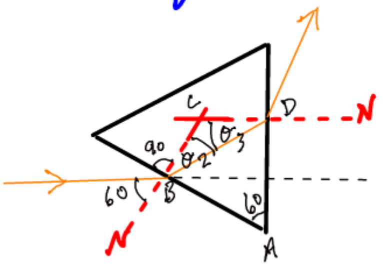
$$n = \sqrt{\left(\frac{2}{\sqrt{3}}sin\theta + \frac{1}{2}\right)^2 + \frac{3}{4}} \quad \text{(A4)} \quad \Big\}\ n(\sin\theta)$$



We use this equation to calculate index of refraction (n) values for our measured angles. We then use the n values to calculate the wavelength using values relating the index of refraction to wavelength for the prism (provided by the supplier of the prism) (see Table III under tab Append 1B).

n(sin θ) goes into eq'n on next page to get Intensity % as a function of λ

**Checking derivation**



$n\sin\theta_3 = n\sin(60° - \theta_2)$ step $\Rightarrow \theta_3 = 60° - \theta_2$

trapezoid ABCD has 360° total angle: use to find angle c interior to "C" A

$360° = 60 + 90 + 90 + \angle c \Rightarrow \angle c = 120°$

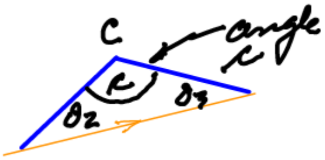
little triangle: $180° = \angle c + \theta_2 + \theta_3 \Rightarrow \theta_2 + \theta_3 = 60°$

$\therefore \theta_3 = 60° - \theta_2$ ✓

$\therefore n\sin\theta_3 = n\cos\theta_2\sin 60° - \cos 60°\sin 60°$ + rearrange

$$n\cos\theta_2 = \frac{n\sin\theta_3}{\sin 60} + \frac{\cos 60°\cancel{\sin 60°}}{\cancel{\sin 60°}}$$

corrected $A_3$ → $n\cos\theta_2 = \dfrac{\sin\theta}{\sin 60} + \cos 60°$ ✓ which Pasco has sin60° end



Squaring $A_1$: $n^2\sin^2\theta_2 = (\sin 60)^2$

" $A_3$: $n^2\cos^2\theta_2 = \left(\dfrac{\sin\theta}{\sin 60} + \cos 60\right)^2$

adding $n^2(\sin^2\theta_2 + \cos^2\theta_2) = \left(\dfrac{\sin\theta}{\sin 60} + \cos 60\right)^2 + (\sin 60)^2$

$$n^2 = \left(\frac{\sin\theta}{\sin 60} + \cos 60\right)^2 + (\sin 60)^2$$

$\sin 60° = \frac{2}{\sqrt{3}}$; $\cos 60° = \frac{1}{2}$

$$n^2 = \left(\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

and: $n = \sqrt{\left(\frac{2}{\sqrt{3}}sin\theta + \frac{1}{2}\right)^2 + \frac{3}{4}}$ (A4) ✓

**Despite error in derivation, the correct #, the 1/2, was put into formula under the √.**

Note the graph below has a range of λ from 400nm to ~2500 nm. The equation developed (A5) applies to this range of λ and not much outside it. This also means the range of Table angle is similarly limited in range ⇒ The data acquired will need to truncated.

(more on this on next page)

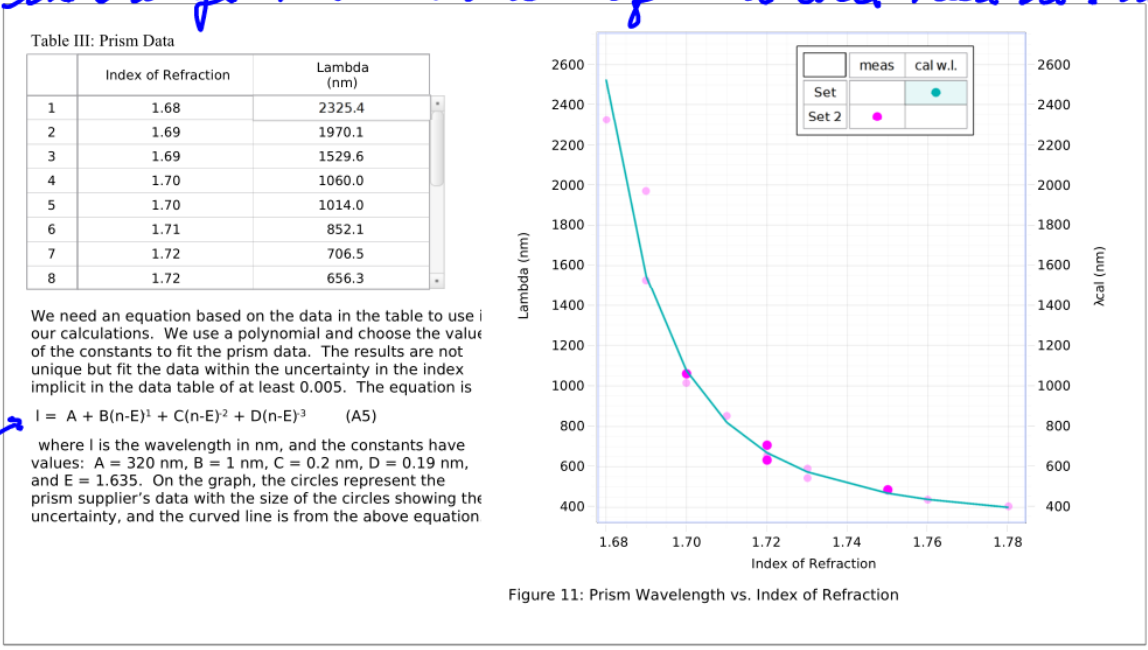


Table III: Prism Data

| | Index of Refraction | Lambda (nm) |
|---|---|---|
| 1 | 1.68 | 2325.4 |
| 2 | 1.69 | 1970.1 |
| 3 | 1.69 | 1529.6 |
| 4 | 1.70 | 1060.0 |
| 5 | 1.70 | 1014.0 |
| 6 | 1.71 | 852.1 |
| 7 | 1.72 | 706.5 |
| 8 | 1.72 | 656.3 |

We need an equation based on the data in the table to use i our calculations. We use a polynomial and choose the value of the constants to fit the prism data. The results are not unique but fit the data within the uncertainty in the index implicit in the data table of at least 0.005. The equation is

λ =

$$l = A + B(n-E)^{1} + C(n-E)^{-2} + D(n-E)^{-3} \qquad (A5)$$

where l is the wavelength in nm, and the constants have values: A = 320 nm, B = 1 nm, C = 0.2 nm, D = 0.19 nm, and E = 1.635. On the graph, the circles represent the prism supplier's data with the size of the circles showing the uncertainty, and the curved line is from the above equation

Figure 11: Prism Wavelength vs. Index of Refraction

So it appears we have: *

λ = 320 nm +

tells xmgrace angle is in degrees not radians

1nm* (sqrt((1.1547* sin(g0.s#.x deg) +0.5)^2+0.75)

−1.635)^-1

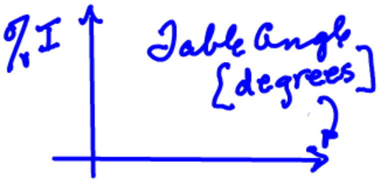
+ 0.2nm* (sqrt((1.1547* sin(g0.s#.x deg) +0.5)^2+0.75)

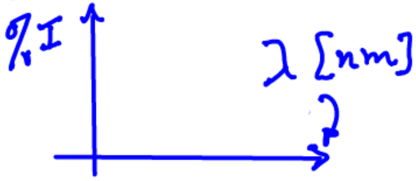
−1.635)^-2

+ 0.19nm* (sqrt((1.1547* sin(g0.s#.x deg) +0.5)^2+0.75)

−1.635)^-3

This is entered into xmgrace to convert the x-axis to λ [nm] from Table Angle in degrees. So we move from:





Truncating data will likely need to be done to get sensible graphs for %I as a function of Table Angle [degrees]. This happens because we can swing the table to angles for which the prism + prism-formula don't refract or function

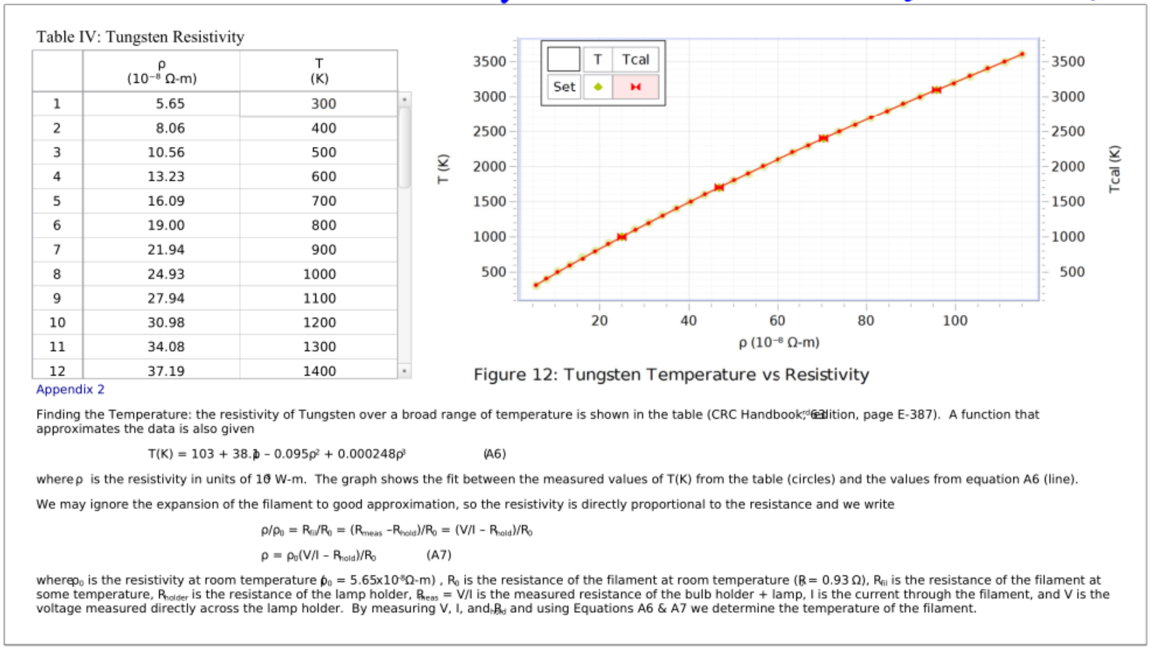


Table IV: Tungsten Resistivity

| | ρ (10⁻⁸ Ω-m) | T (K) |
|---|---|---|
| 1 | 5.65 | 300 |
| 2 | 8.06 | 400 |
| 3 | 10.56 | 500 |
| 4 | 13.23 | 600 |
| 5 | 16.09 | 700 |
| 6 | 19.00 | 800 |
| 7 | 21.94 | 900 |
| 8 | 24.93 | 1000 |
| 9 | 27.94 | 1100 |
| 10 | 30.98 | 1200 |
| 11 | 34.08 | 1300 |
| 12 | 37.19 | 1400 |

Figure 12: Tungsten Temperature vs Resistivity

Appendix 2

Finding the Temperature: the resistivity of Tungsten over a broad range of temperature is shown in the table (CRC Handbook, 63rd edition, page E-387). A function that approximates the data is also given

$$T(K) = 103 + 38.1\rho - 0.095\rho^2 + 0.000248\rho^3 \qquad (A6)$$

where ρ is the resistivity in units of $10^{-8}$ W-m. The graph shows the fit between the measured values of T(K) from the table (circles) and the values from equation A6 (line).

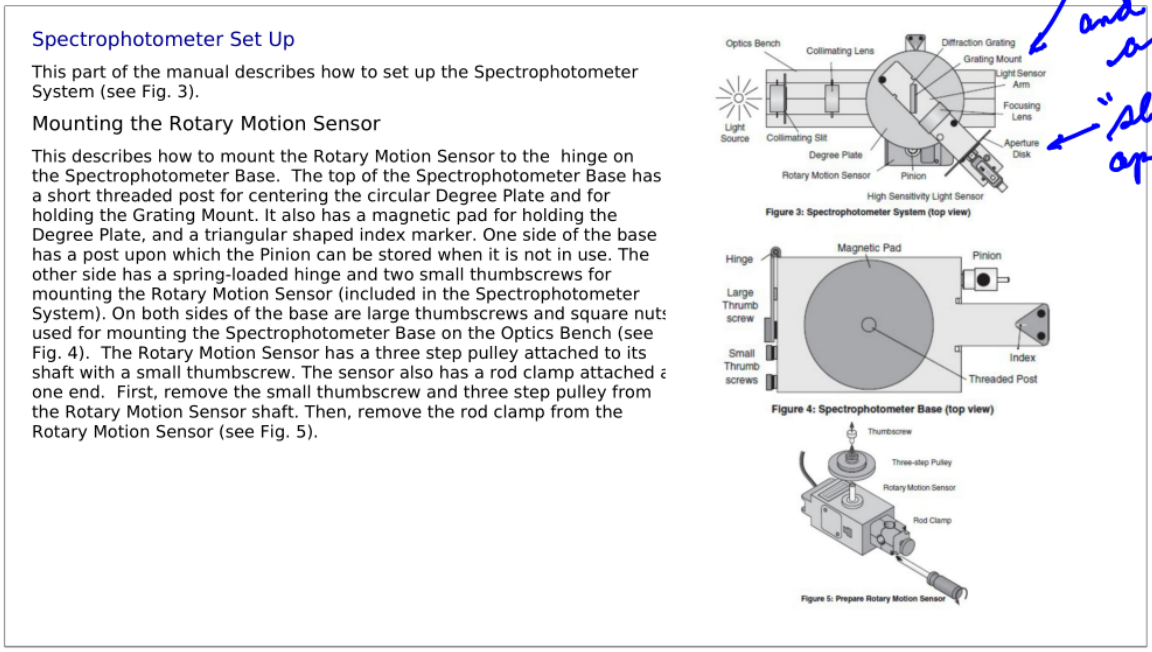
We may ignore the expansion of the filament to good approximation, so the resistivity is directly proportional to the resistance and we write

$$\rho/\rho_0 = R_{fil}/R_0 = (R_{meas} - R_{hold})/R_0 = (V/I - R_{hold})/R_0$$

$$\rho = \rho_0(V/I - R_{hold})/R_0 \qquad (A7)$$

where $\rho_0$ is the resistivity at room temperature ($\rho_0 = 5.65\text{x}10^{-8}\Omega$-m) , $R_0$ is the resistance of the filament at room temperature ($R_0 = 0.93\ \Omega$), $R_{fil}$ is the resistance of the filament at some temperature, $R_{holder}$ is the resistance of the lamp holder, $R_{meas}$ = V/I is the measured resistance of the bulb holder + lamp, I is the current through the filament, and V is the voltage measured directly across the lamp holder. By measuring V, I, and $R_{hold}$ and using Equations A6 & A7 we determine the temperature of the filament.

2018: leave this step out. It is a method of getting the filament temperature for verification with the temperature derived from the BB curve(s).



## Spectrophotometer Set Up

This part of the manual describes how to set up the Spectrophotometer System (see Fig. 3).

### Mounting the Rotary Motion Sensor

This describes how to mount the Rotary Motion Sensor to the hinge on the Spectrophotometer Base. The top of the Spectrophotometer Base has a short threaded post for centering the circular Degree Plate and for holding the Grating Mount. It also has a magnetic pad for holding the Degree Plate, and a triangular shaped index marker. One side of the base has a post upon which the Pinion can be stored when it is not in use. The other side has a spring-loaded hinge and two small thumbscrews for mounting the Rotary Motion Sensor (included in the Spectrophotometer System). On both sides of the base are large thumbscrews and square nut used for mounting the Spectrophotometer Base on the Optics Bench (see Fig. 4). The Rotary Motion Sensor has a three step pulley attached to its shaft with a small thumbscrew. The sensor also has a rod clamp attached one end. First, remove the small thumbscrew and three step pulley from the Rotary Motion Sensor shaft. Then, remove the rod clamp from the Rotary Motion Sensor (see Fig. 5).

Figure 3: Spectrophotometer System (top view)

Figure 4: Spectrophotometer Base (top view)

Figure 5: Prepare Rotary Motion Sensor

mistake: should be prism and "other" angle

"slit aperture"

# METHOD

RMS = Rotary Motion Sensor:

① Remove black mounting h/w from RMS sensor using provided hex-key. Store hex key in RMS for later use.

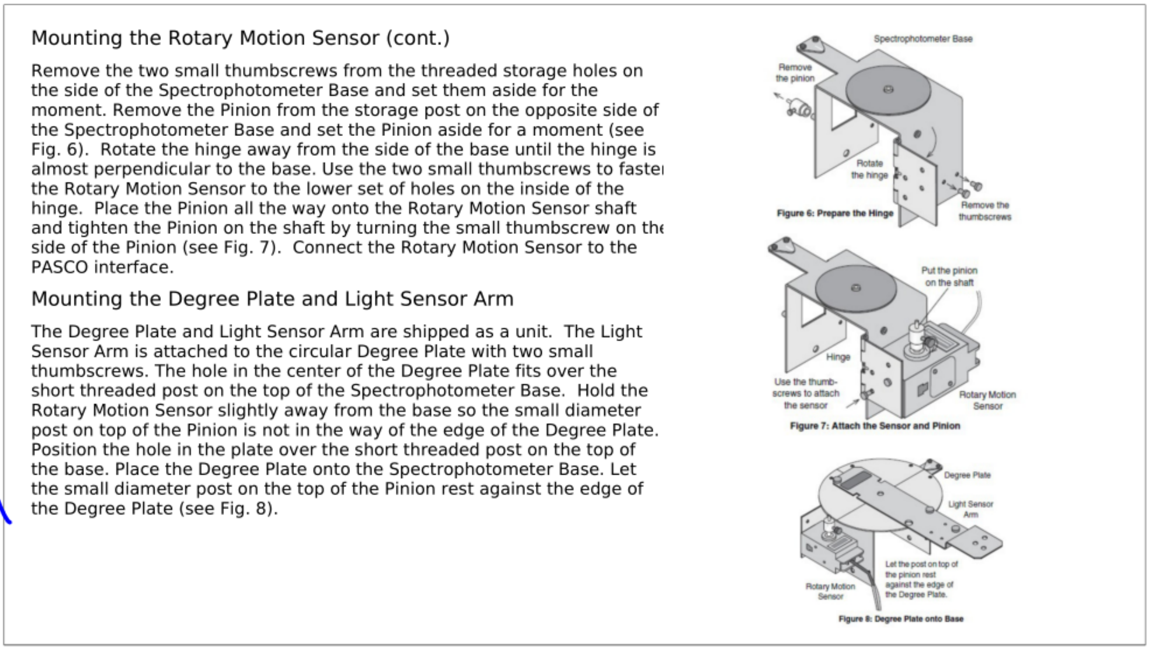
② Remove 3-step pulley + replace knurled retaing screw in RMS shaft.

③ RMS to spring-loaded plate:

a) Use lower pair of holes: gives proper distance and alignment for pinion

b) May require 2 small washers as mounting screws could bottom-out in RMS.

④ See further instructions on next page.



### Mounting the Rotary Motion Sensor (cont.)

Remove the two small thumbscrews from the threaded storage holes on the side of the Spectrophotometer Base and set them aside for the moment. Remove the Pinion from the storage post on the opposite side of the Spectrophotometer Base and set the Pinion aside for a moment (see Fig. 6). Rotate the hinge away from the side of the base until the hinge is almost perpendicular to the base. Use the two small thumbscrews to faste the Rotary Motion Sensor to the lower set of holes on the inside of the hinge. Place the Pinion all the way onto the Rotary Motion Sensor shaft and tighten the Pinion on the shaft by turning the small thumbscrew on the side of the Pinion (see Fig. 7). Connect the Rotary Motion Sensor to the PASCO interface.

### Mounting the Degree Plate and Light Sensor Arm

The Degree Plate and Light Sensor Arm are shipped as a unit. The Light Sensor Arm is attached to the circular Degree Plate with two small thumbscrews. The hole in the center of the Degree Plate fits over the short threaded post on the top of the Spectrophotometer Base. Hold the Rotary Motion Sensor slightly away from the base so the small diameter post on top of the Pinion is not in the way of the edge of the Degree Plate. Position the hole in the plate over the short threaded post on the top of the base. Place the Degree Plate onto the Spectrophotometer Base. Let the small diameter post on the top of the Pinion rest against the edge of the Degree Plate (see Fig. 8).

Figure 6: Prepare the Hinge

Figure 7: Attach the Sensor and Pinion

Figure 8: Degree Plate onto Base

likely complete already

METHOD con't

⑤ Alignment:

* Once constructed one must know that 0° is really 0°. This means location of
light source
collimating slits
collimating lens
prism
focusing lens
slit aperture in front of broad spectrum sensor
broad spectrum sensor.

must all be positioned so that the "white" light from the source hits the slit aperture when the table is set to zero degrees.

* An effective way to move the beam of light for this step is to slightly loosen + the collimating slits and watch where the light lands on the slit aperture